Inspired Learning

1.

	$f(x) = 2x^3 - 7x^2 + 4x + 4$		
	$f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$	Attempts $f(2)$ or $f(-2)$	M1
(a)	$= 0$, and so $(x-2)$ is a factor.	$f(\mathbf{2}) = 0$ with no sign or substitution errors ($2(2)^3 - 7(2)^2 + 4(2) + 4 = 0$ is sufficient) **and for conclusion.** Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion **but not = 0 just underlined and not hence (2 or f(2)) is a factor**. Note also that a conclusion can be implied from a preamble, eg: "If $f(2) = 0$, $(x-2)$ is a factor…."	A1
	Note: Long division scores no marks in part (a). The factor theorem is required.		
			[2]
(b)	$f(x) = \{(x-2)\}(2x^2 - 3x - 2)$	M1: Attempts long division by $(x-2)$ or other method using $(x-2)$, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." A1: $(2x^2 - 3x - 2)$	M1 A1
	$= (x-2)(x-2)(2x+1)$ or $(x-2)^2(2x+1)$ or equivalent e.g. $= 2(x-2)(x-2)(x+\frac{1}{2})$ or $2(x-2)^2(x+\frac{1}{2})$	dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). **This is dependent** on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the **factors**. A1: cao – needs all three factors **on one line**. Ignore following work (such as a solution to a quadratic equation.)	dM1 A1
	Note $= (x-2)(\frac{1}{2}x - 1)(4x+2)$ would lose the last mark as it is not **fully** factorised		
	For correct answers only award full marks in (b)		
			[4]
			Total 6

2.

	Solution	Marks	Total	Comments
(a)	$f(-3) = (-3)^3 - 4\times(-3) + 15$	M1		f(–3) attempted **not** long division
	$f(-3) = -27 + 12 + 15$			
	$= 0 \Rightarrow x+3$ is a factor	A1	2	shown = 0 plus statement
(ii)	Quadratic factor $(x^2 - 3x + 5)$	M1		$-3x$ or $+5$ term by inspection *or* full long division attempt
	$(f(x) =)\ (x+3)(x^2 - 3x + 5)$	A1	2	must see correct product

3.

12	(i)		$f(1) = 1 - 1 + 1 + 9 - 10$ [= 0]	B1	allow for correct division of f(x) by (x – 1) showing there is no remainder, or for $(x-1)(x^3 + x + 10)$ found, showing it 'works' by multiplying it out	condone $1^4 - 1^3 + 1^2 + 9 - 10$
			attempt at division by (x – 1) as far as $x^4 - x^3$ in working	M1	allow equiv for (x + 2) as far as $x^4 + 2x^3$ in working or for inspection with at least two terms of cubic factor correct	eg for inspection, M1 for two terms right and two wrong
			correctly obtaining $x^3 + x + 10$	A1 [3]	or $x^3 - 3x^2 + 7x - 5$	if M0 and this division / factorising is done in part (ii) or (iii), allow SC1 if correct cubic obtained there; attach the relevant part to (i) with a formal chain link if not already seen in the image zone for (i)

12	(ii)		[g(–2) =] $-8 - 2 + 10$ or $f(-2) = 16 + 8 + 4 - 18 - 10$	M1	[in this scheme $g(x) = x^3 + x + 10$] allow M1 for correct trials with at least two values of x (other than 1) using g(x) or f(x) or $x^3 - 3x^2 + 7x - 5$ (may allow similar correct trials using division or inspection)	eg $f(2) = 16 - 8 + 4 + 18 - 10$ or 20; $f(3) = 81 - 27 + 9 + 27 - 10$ or 80; $f(0) = -10$; $f(-1) = 1 + 1 + 1 - 9 - 10$ or -16; No ft from wrong cubic 'factors' from (i)
			$x = -2$ isw	A1 [2]	allow these marks if already earned in (i)	NB factorising of $x^3 + x + 10$ or $x^3 - 3x^2 + 7x - 5$ in (ii) earns credit for (iii) [annotate with a yellow line in both parts to alert you – the image zone for (iii) includes part (ii)]

Question		Answer	Marks	Guidance	
	(iii)	attempted division of $x^4 + x + 10$ by $(x + 2)$ as far as $x^3 + 2x^2$ in working	M1	or $x^3 - 3x^2 + 7x - 5$ by $(x - 1)$ as far as $x^3 - x^2$ in working or inspection with at least two terms of quadratic factor correct	alt method: allow M1 for attempted division of quartic by $x^2 + x - 2$ as far as $x^4 + x^3 - 2x^2$ in working, or inspection etc
		correctly obtaining $x^2 - 2x + 5$	A1	allow these first 2 marks if this has been done in (ii), even if not used here	
		use of $b^2 - 4ac$ with $x^2 - 2x + 5$	M1	may be in attempt at formula (ignore rest of formula)	or completing square form attempted or attempt at calculus or symmetry to find min pt NB M0 for use of $b^2 - 4ac$ with cubic factor etc
		$b^2 - 4ac = 4 - 20$ [$= -16$]	A1	may be in formula;	or $(x - 1)^2 + 4$ or min = (1, 4)
		so only two real roots[of f(x)] [and hence no more linear factors]	A1	or no real roots of $x^2 - 2x + 5 = 0$; allow this last mark if clear use of $x^2 - 2x + 5 = 0$, even if error in $b^2 - 4ac$, provided result negative, but no ft from wrong factor if last M1 not earned, allow SC1 for stating that the only factors of 5 are 1 and 5 and reasoning eg that $(x - 1)(x - 5)$ and $(x + 1)(x + 5)$ do not give $x^2 - 2x + 5$ [hence $x^2 - 2x + 5$ does not factorise]	or $(x - 1)^2 + 4$ is always positive so no real roots [of $(x - 1)^2 + 4 = 0$] [and hence no linear factors] or similar conclusion from min pt
			[5]		

4.	(i)	$f(1) = 1$ $f(-1) = 21$ $f(2) = 0$, hence $(x - 2)$ is a factor	M1		Attempt use of factor theorem at least once
			A1	2	Obtain factor of $(x - 2)$

(ii)	$f(x) = (x-2)(x^2+3x-5)$	**M1**		Attempt complete division by a linear factor, or equivalent ie inspection or coefficient matching
	$x = \dfrac{-3 \pm \sqrt{29}}{2}$ or $x = 2$	**A1**		Obtain x^2+3x+c or x^2+bx-5
		A1		Obtain x^2+3x-5
		M1		Attempt to solve quadratic equation
		A1		Obtain $\frac{1}{2}(-3 \pm \sqrt{29})$
		B1	**6**	State 2 as root, at any point
			8	

5.

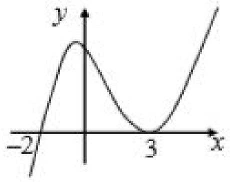
4. (a)	$f(x) = -4x^3 + ax^2 + 9x - 18$		
	$f(2) = -32 + 4a + 18 - 18 = 0$	Attempts $f(2)$ or $f(-2)$	M1
	$\Rightarrow 4a = 32 \Rightarrow a = 8$	cso	A1
			[2]
(a) Way 2	$f(x) = (x-2)(px^2+qx+r)$		
	$= px^3 + (q-2p)x^2 + (r-2q)x - 2r$		
	$r = 9 \Rightarrow q = 0$ also $p = -4 \therefore a = -2p = 8$	Compares coefficients leading to $-2p = a$	M1
	$a = 8$	cso	A1
(a) Way 3	$(-4x^3 + ax^2 + 9x - 18) \div (x-2)$		
	$Q = -4x^2 + (a-8)x + 2a - 7$ $R = 4a - 32$	Attempt to divide $\pm f(x)$ by $(x-2)$ to give a quotient at least of the form $\pm 4x^2 + g(a)x$ and a remainder that is a function of a	M1
	$4a - 32 = 0 \Rightarrow a = 8$	cso	A1

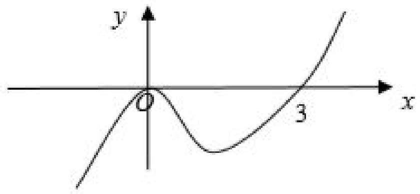
(b)	$\mathrm{f}(x) = (x-2)(-4x^2+9)$	Attempts long division or other method, to obtain $(-4x^2 \pm ax \pm b)$, $b \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection."	M1
	$= (x-2)(3-2x)(3+2x)$ or equivalent e.g. $= -(x-2)(2x-3)(2x+3)$ or $= (x-2)(2x-3)(-2x-3)$	dM1: A ***valid*** attempt to factorise their quadratic – see General Principles. **This is dependent** on the previous method mark being awarded, but there must have been no remainder. A1: cao – must have all 3 factors on the same line. Ignore subsequent work (such as a solution to a quadratic equation.)	dM1A1
			[3]
(c)	$\mathrm{f}\left(\frac{1}{2}\right) = -4\left(\frac{1}{8}\right) + 8\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) - 18 = -12$	Attempts $\mathrm{f}\left(\frac{1}{2}\right)$ or $\mathrm{f}\left(-\frac{1}{2}\right)$ Allow A1ft for the correct numerical value of $\frac{\text{their } a}{4} - 14$	M1A1ft
			[2]
(c) Way 2	$\pm\left(-4x^3 + 8x^2 + 9x - 18\right) \div (2x-1)$		
	$Q = -2x^2 + 3x + 6$ $R = -12$	M1: Attempt long division to give a remainder that is independent of x A1: Allow A1ft for the correct numerical value of $\frac{\text{their } a}{4} - 14$.	M1A1ft

6.	(i)		f(–3) used	M1
			$-54 - 27 + 69 + 12$ [= 0] isw	A1
			attempt at division by $(x + 3)$ as far as $2x^3 + 6x^2$ in working	M1
			correctly obtaining $2x^2 - 9x + 4$	A1
			factorising the correct quadratic factor	M1
			$(2x - 1)(x - 4)[(x + 3)]$ isw	A1
				[6]

(ii)		sketch of cubic right way up, with two turning points	B1	0 if stops at x-axis ignore graph of $y = 4x + 12$
		values of intns on x axis shown, correct (-3, 0.5 and 4) or ft from their factors or roots in (i)	B1	on graph or nearby in this part mark intent for intersections with both axes
		12 marked on y-axis	B1 **[3]**	or $x = 0$, $y = 12$ seen in this part if consistent with graph drawn
(iii)		$2x^3 - 3x^2 - 23x + 12 = 4x + 12$ oe	M1	or ft their factorised f(x)
		$2x^3 - 3x^2 - 27x$ [= 0]	A1	after equating, allow A1 for cancelling ($x + 3$) factor on both sides and obtaining $2x^2 - 9x$ [= 0]
		[x]($2x - 9$)($x + 3$) [= 0]	M1	for linear factors of correct cubic, giving two terms correct or for quadratic formula or completing square used on correct quadratic $2x^2 - 3x - 27 = 0$, condoning one error in formula etc;
		[x =] 0, -3 and 9/2 oe	A1 **[4]**	need not be all stated together

7.

5(a)	$p(-1) = (-1)^3 - 4\times(-1)^2 - 3(-1) + 18$	M1		p(–1) attempted **not** long division
	$(= -1 - 4 + 3 + 18) = 16$	A1	2	
(b)(i)	$p(3) = 3^3 - 4\times 3^2 - 3\times 3 + 18$	M1		p(3) attempted **not** long division
	$p(3) = 27 - 36 - 9 + 18 = 0 \Rightarrow x - 3$ is a factor	A1	2	shown = 0 plus statement
(ii)	Quadratic factor $(x^2 - x + c)$ or $(x^2 + bx - 6)$	M1		$-x$ or -6 term by inspection *or* full long division by $x - 3$ *or* comparing coefficients *or* p(–2) attempted
	Quadratic factor $(x^2 - x - 6)$	A1		correct quadratic factor (or $x+2$ shown to be factor by Factor Theorem)
	$[p(x) =]\ (x-3)(x-3)(x+2)$	A1	3	or $[p(x) =]\ (x-3)^2(x+2)$ must see product of factors
(c)		M1		cubic curve with one maximum and one minimum
		A1		meeting x-axis at –2 and touching x-axis at 3
	Final A1 is dependent on previous A1 and can be withheld if curve has very poor curvature beyond $x = 3$, V shape at $x = 3$ etc	A1	3	graph as shown, going beyond $x = -2$ but condone max on or to right of y-axis
	Total		**10**	

8. (a)		M1		cubic curve touching at O – one max, one min (may have minimum at O)
		A1		shape roughly as shown crossing positive x-axis
		A1	3	3 marked and correct curvature for $x < 0$ and $x > 3$
(b)(i)	$p(4) = 4^2(4-3)+20$	M1		p(4) attempted **or** full long division as far as remainder term
	(Remainder) $= 36$	A1	2	
(ii)	$p(-2) = (-2)^2(-2-3)+20$	M1		p(–2) attempted NOT long division
	$= 4\times(-5)+20 = 0$ *or* $-20+20=0$ therefore $(x+2)$ is a factor	A1	2	working showing that p(–2) = 0 and statement
(iii)	x^2+bx+c *with* $b=-5$ **or** $c=10$	M1		by inspection
	$(x+2)(x^2-5x+10)$	A1	2	must see product
(iv)	Discriminant of "their" quadratic $=(-5)^2-4\times 10$	M1		be careful that cubic coefficients are not being used
	$-15<0$ so quadratic has no real roots	A1cso		
	(only real root is) –2	B1	3	independent of previous marks
	Total		**12**	

(a)	Award **M1** for clear ***intention*** to touch at *O* Second **A1:** allow curve becoming straight but withhold if wrong curvature in 1st or 3rd quadrants.
(b) (i)	May expand cubic as x^3-3x^2+20 Do not apply ISW for eg " p(4) = 36, therefore remainder is – 36"
(ii)	Minimum required for statement is "so factor" Powers of –2 must be evaluated: **Example** "p(–2) = –8–12+20 = 0 therefore factor" scores **M1 A1** Statement may appear first : **Example** " x+2 is factor if p(–2) = 0 & p(–2) = –8–12+20 = 0" scores **M1 A1** However, **Example** "$p(-2)=(-2)^2(-2-3)+20 = 0$ therefore x+2 is a factor" scores **M1 A0**
(iii)	**M1** may also be earned for a full long division attempt, or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients .
(iv)	Accept "$b^2-4ac = 25-40<0$ so no real roots" for **M1 A1cso** Discriminant may appear within the quadratic equation formula " $\sqrt{25-40}$ " for **M1**

9.

f(2) = 18 seen or used	M1	or long division oe as far as obtaining a remainder (ie not involving x) and equating that remainder to 18 (there may be errors along the way)	
$32 + 2k - 20 = 18$ oe	A1	after long division: $2(k + 16) - 20 = 18$ oe	A0 for just 2^5 instead of 32 unless 32 implied by further work
$[k =] 3$	A1 [3]		

10.

use of $f(2)$	M1	2 substituted in $f(x)$ or $f(2) = 42$ seen or correct division of $4x^3 + kx + 6$ by $x - 2$ as far as obtaining $4x^2 + 8x + (k + 16)$ oe [may have $4x^2 + 8x + 18$]	
$4 \times 2^3 + 2k + 6 = 42$	M1	or $6 + 2(k + 16) = 42$ oe or finding (usually after division) that the constant term is 36 and then working with the x term to find k eg $kx + 16x = 18x$	
$k = 2$	A1		
$[x =] -1$	A1	as their answer, not just a trial; A0 for just $f(-1) = 0$ with no further statement; A0 if confusion between roots and factors in final statement eg '$x + 1$ is a root', even if they also state $x = -1$	accept with no working since it can be found by inspection
	[4]		

11.

(a)	$p(-2) = (-2)^3 + (-2)^2c + (-2)d - 12$	M1		p(−2) attempted *or* long division by $x+2$ as far as remainder
	'their' $-8 + 4c - 2d - 12 = -150$	m1		putting expression for remainder = −150
	$\Rightarrow 2c - d + 65 = 0$	A1cso	3	**AG** terms all on one side in any order (check that there are no errors in working)
(b)	$p(3) = 3^3 + 3^2c + 3d - 12$	M1		p(3) attempted *or* long division by $x-3$ as far as remainder
	$9c + 3d + 15 = 0$	A1	2	any correct equation with terms collected eg $3c + d = -5$
(c)	$\left.\begin{array}{l}2c - d + 65 = 0\\ 3c + d + 5 = 0\end{array}\right\} \Rightarrow 5c = -70$	M1		Elimination of c or d
	$\Rightarrow c = -14$, $d = 37$ OE	A1 A1	3	value of c ***or*** d correct unsimplified; both c and d correct unsimplified

12.	(a)	$f(x) = x^4 + x^3 + 2x^2 + ax + b$	
		Attempting f(1) or f(−1).	M1
		$f(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$ (as required) **AG**	A1 * **cso** (2)
	(b)	Attempting f(−2) or f(2).	M1
		$f(-2) = \underline{16 - 8 + 8 - 2a + b = -8}$ $\{\Rightarrow -2a + b = -24\}$	A1
		Solving both equations simultaneously to get as far as $a = ...$ or $b = ...$	dM1
		Any one of $a = 9$ or $b = -6$	A1
		Both $a = 9$ and $b = -6$	A1 **cso** (5) **[7]**

(a)	M1 for attempting either f(1) or f(−1). A1 for applying f(1), setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).
(b)	M1: attempting either f(−2) or f(2). A1: correct underlined equation in a and b; eg $\underline{16 - 8 + 8 - 2a + b = -8}$ or equivalent, eg $-2a + b = -24$. dM1: an attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark. A1: any one of $a = 9$ or $b = -6$. A1: both $a = 9$ and $b = -6$ and a correct solution only.
	Alternative Method of Long Division: (a) M1 for long division by $(x - 1)$ to give a remainder in a and b which is independent of x. A1 for {Remainder =} $b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given.) (b) M1 for long division by $(x + 2)$ to give a remainder in a and b which is independent of x. A1 for {Remainder =} $\underline{b - 2(a - 8) = -8}$ $\{\Rightarrow -2a + b = -24\}$. Then dM1A1A1 are applied in the same way as before.

13.

$f(2) = 8 + 2a - 6 + 2b = 0$ $g(2) = 24 + 4 + 10a + 4b = 0$	M1	Attempt at least one of f(2), g(2)	Allow for substituting $x = 2$ into either equation – no need to simplify at this stage. Division – complete attempt to divide by $(x - 2)$. Coeff matching - attempt all 3 coeffs of quadratic factor.
	M1	Equate at least one of f(2) and g(2) to 0	Just need to equate their substitution attempt to 0 (but just writing eg f(2) = 0 is not enough). It could be implied by later working, even after attempt to solve equations. Division - equating their remainder to 0. Coeff matching – equate constant terms.
$2a + 2b = -2$, $5a + 2b = -14$	A1	Obtain two correct equations in a and b	Could be unsimplified equations. Could be $8a + 2b = -26$ (from f(2) = g(2)).
hence $3a = -12$	M1	Attempt to find a (or b) from two simultaneous eqns	Equations must come from attempts at two of f(2) = 0, g(2) = 0, f(2) = g(2). M1 is awarded for eliminating a or b from 2 sim eqns – allow sign slips only. Most will attempt a first, but they can also gain M1 for finding b from their simultaneous equations.
so $a = -4$ **AG**	A1	Obtain $a = -4$, with necessary working shown	If finding b first, then must show at least one line of working to find a (unless earlier shown explicitly eg $a = -1 - b$).
$b = 3$	A1	Obtain $b = 3$	Correct working only
	[6]		**SR Assuming $a = -4$** **Either use this scheme, or the original, but don't mix elements from both** M1 Attempt either f(2) or g(2) M1 Equate f(2) or g(2) to 0 (also allow f(2) = g(2)) A1 Obtain $b = 3$ A1 Use second equation to confirm $a = -4$, $b = 3$

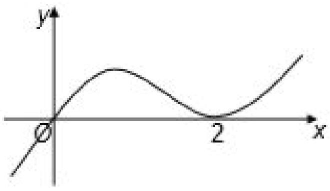
$f(x) = (x-2)(x^2+2x-3)$ $= (x-2)(x+3)(x-1)$	M1	Attempt full division of their f(x) by $(x-2)$ Could also be for full division attempt by $(x-1)$ or $(x+3)$ if identified as factors	Must be using $f(x) = x^3 - 7x + k$. Must be complete method – ie all 3 terms attempted. Long division – must subtract lower line (allow one slip). Inspection – expansion must give at least three correct terms of their cubic. Coefficient matching – must be valid attempt at all 3 quadratic coeffs, considering all relevant terms each time. Factor theorem – must be finding 2 more factors / roots.
	A1	Obtain x^2 and at least one other correct term, from correct f(x)	Could be middle or final term depending on method. Coeff matching – allow for stating values eg $A = 1$ etc. Factor theorem – state factors of $(x+3)$ and $(x-1)$.
	A1	Obtain $(x-2)(x+3)(x-1)$	Must be seen as a product of three linear factors. Answer only gains all 3 marks.
$g(x) = (x-2)(3x^2+7x-6)$ $= (x-2)(x+3)(3x-2)$ **OR** $g(1) = -4,\ g(-3) = 0$	M1	Attempt to verify two common factors	Possible methods are: Factorise g(x) completely – f(x) must have been factorised. Find quadratic factor of g(x) and identify $x = -3$ as root. Test their roots of f(x) in g(x). Just stating eg $g(-3) = 0$ is not enough – working required. If f(x) hasn't been factorised, allow M1 for using factor thm on both functions to find common factor, or for factorising g(x) and testing roots in f(x).
Hence common factor of $(x+3)$	A1 [5]	Identify $(x+3)$ as a common factor	Just need to identify $(x+3)$ - no need to see $(x-2)$ or to explicitly state 'two common factors'. Need to see $(x+3)$ as factor of g(x) – just showing $g(-3) = 0$ and then stating 'common factor' is not enough. CWO (inc A0 for $g(x) = (x-2)(x+3)(x-{}^2/_3)$). If using factor thm, no need to find g(1) if g(-3) done first. Just stating $(x+3)$ with no supporting evidence is M0A0. A0 if referring to -3 (and 2) as 'factors'. A0 if additional incorrect factor given.

14.

(i)	$f(3) = 54 + 27 - 51 + 6$ $= 36$	**M1** **A1** [2]	Attempt f(3) Obtain 36	Allow equiv methods as long as remainder is attempted A0 if answer subsequently stated as -36 ie do not isw

(ii)	$f(x) = (x-2)(2x^2+7x-3)$	B1	State or imply that $(x-2)$ is a factor	Just stating this is enough for B1, even if not used Could be implied by attempting division, or equiv, by $(x-2)$
		M1	Attempt full division, or equiv, by $(x \pm 2)$	Must be complete method – ie all three terms attempted If long division then must subtract lower line (allow one slip); if inspection then expansion must give correct first and last terms and also one of the two middle terms of the cubic; if coefficient matching then must be valid attempt at all 3 quadratic coeffs, considering all relevant terms each time Allow M1 for valid division attempt by $(x+2)$
		A1	Obtain $2x^2$ and at least one other correct term	If coeff matching then allow for stating values eg $A = 2$ etc
		A1 [4]	Obtain $(x-2)(2x^2+7x-3)$	Must be stated as a product
(iii)	$b^2 - 4ac = 73$ > 0 hence 3 roots	M1	Attempt explicit numerical calculation to find number of roots of quadratic	Could attempt discriminant (allow $b^2 \pm 4ac$), or could use full quadratic formula to attempt to find the roots themselves (implied by stating decimal roots); M0 for factorising unless their incorrect quotient could be factorised M0 for '3 roots as positive discriminant' but no evidence
		A1ft [2]	State 3 roots (√ their quotient) Condone no explicit check for repeated roots	Sufficient working must be shown, and all values shown must be correct Discriminant needs to be 73 (allow $7^2 - 4(2)(-3)$) Quadratic formula must be correct, though may not necessarily be simplified as far as $\frac{1}{4}(-7 \pm \sqrt{73})$ Need to state no. of roots – just listing them is not enough **SR**: if a conclusion is given in part (iii) then allow evidence from part (ii) eg finding actual roots

15.

(a)(i)		M1 A1 A1	 3	cubic curve with one max and one min (either way up) curve touching positive x-axis (either way up) correct graph passing through O and touching x-axis at 2
(ii)	$x(x^2-4x+4)=3$ $\Rightarrow x^3-4x^2+4x-3=0$	B1	1	AG (must have = 0)

(b)(i)	$p(-1) = (-1)^3 - 4(-1)^2 + 4(-1) - 3$ $(= -1 - 4 - 4 - 3)$	M1		p(−1) attempted (condone one slip) or full long division to remainder
	$= -12$	A1	2	must indicate remainder $= -12$ if long division used
(ii)	$p(3) = 3^3 - 4\times3^2 + 4\times3 - 3$	M1		p(3) attempted (condone one slip) NOT long division
	$p(3) = 27 - 36 + 12 - 3$ $p(3) = 0 \Rightarrow x - 3$ is factor	A1	2	shown = 0 **plus statement**
(iii)	Either $b = -1$ (coefficient of x correct) or $c = 1$ (constant term correct)	M1		allow M1 for full attempt at long division or comparing coefficients if neither b nor c is correct
	$p(x) = (x-3)(x^2 - x + 1)$	A1	2	
(c)	Discriminant of 'their quadratic' $= (-1)^2 - 4$	M1		numerical expression must be seen
	Discriminant $= -3$ (or < 0) $\Rightarrow$ no real roots	A1cso		must have correct quadratic and statement and all working correct
	(Only real root is $x =$) 3	B1	3	
	Total		**13**	

16.

. (a)	Either (Way 1) : Attempt f(3) or f(−3) $f(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ *	Or (Way 2): Assume $a = -9$ and attempt f(3) or f(−3) f(3) = 0 so (x -3) is factor	M1 A1 * cso **(2)**
	Or (Way 3): $(2x^3 - 5x^2 + ax + 18) \div (x-3) = 2x^2 + px + q$ where p is a number and q is an expression in terms of a Sets the remainder $18+3a+9 = 0$ and solves to give $a = -9$		M1 A1* cso **(2)**
(b)	Either (Way 1): $f(x) = (x-3)(2x^2 + x - 6)$ $= (x-3)(2x-3)(x+2)$		M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x = -2$ **or** $x = 3/2$ Uses trial or factor theorem to obtain both $x = -2$ **and** $x = 3/2$ Puts three factors together (see notes below) Correct factorisation : $(x-3)(2x-3)(x+2)$ or $(3-x)(3-2x)(x+2)$ or $2(x-3)(x-\frac{3}{2})(x+2)$ oe		M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x-3)(2x-3)(x+2)$ otherwise need working		M1A1M1A1

(a)	M1 for attempting either f(3) or f(−3) – with **numbers substituted into expression** A1 for applying f(3) **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept x = -9) Note that the answer is given in part (a). If they **assume** a = -9 and **verify** by factor theorem or division they must state $(x - 3)$ **is a factor** for A1 (or equivalent such as QED or a tick).
(b)	1st M1: attempting to divide by $(x - 3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by $(3 - x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1st A1: usually for $2x^2 + x - 6$ ... Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M0A0, $(x - 3)(x - \frac{3}{2})(2x + 4)$ is M1A1M1A0, but $2(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M1A1.