Inspired Learning

1.

$4x^3 \to kx^2$ or $2x^{\frac{1}{2}} \to kx^{-\frac{1}{2}}$ (k a non-zero constant)	M1
$12x^2, + x^{-\frac{1}{2}}$......., $(-1 \to 0)$	A1, A1, B1 (4)
	4

2.

Question Number	Scheme	Marks
1. (a)	$f'(x) = 3x^2 + 6x$	B1
	$f''(x) = 6x + 6$	M1, A1cao (3)

Notes cao = correct answer only

1(a)	
Acceptable alternatives include $3x^2 + 6x^1$; $3x^2 + 3\times 2x$; $3x^2 + 6x + 0$ Ignore LHS (e.g. use [whether correct or not] of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$) $3x^2 + 6x + c$ or $3x^2 + 6x$ + constant (i.e. the written word constant) is B0	B1
M1 Attempt to differentiate their $f'(x)$; $x^n \to x^{n-1}$. $x^n \to x^{n-1}$ seen in at least one of the terms. Coefficient of $x^{\cdots}$ ignored for the method mark. $x^2 \to x^1$ and $x \to x^0$ are acceptable.	M1
Acceptable alternatives include $6x^1 + 6x^0$; $3\times 2x + 3\times 2$ $6x + 6 + c$ or $6x + 6$ + constant is A0	A1 cao

3.

i)	$y = 6x^3 + 4x^{-\frac{1}{2}} + 5x$ $\frac{dy}{dx} = 18x^2 - 2x^{-\frac{3}{2}} + 5$	B1 M1 A1 A1 [4]	$\frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$ soi Attempt to differentiate, any term correct Two correct terms Fully correct, no "+c"	
ii)	$\frac{d^2y}{dx^2} = 36x + 3x^{-\frac{5}{2}}$	M1 A1 [2]	Attempt to differentiate their $\frac{dy}{dx}$ cao www in either part	Any term still involving x correct – follow through from their expression for the **M** mark only

4.

Question		Answer	Marks	Guidance	
(i)		$-10x^{-6}$ isw	B1 B1 [2]	for -10 for x^{-6} ignore $+c$ and $y =$	if B0B0 then SC1 for $-5\times2x^{-5-1}$ or better soi
(ii)		$y = x^{1/3}$ soi kx^{n-1} $\frac{1}{3}x^{-\frac{2}{3}}$ isw	B1 M1 A1 [3]	condone $y' = x^{1/3}$ if differentiation follows ft their fractional n ignore $+c$ and $y =$	 allow 0.333 or better

5.

(i)		$f(x) = 6x^{-2} + 2x$ $f'(x) = -12x^{-3} + 2$	M1 A1 B1 [3]	kx^{-3} obtained by differentiation $-12x^{-3}$ $2x$ correctly differentiated to 2	ISW incorrect simplification after correct expression
(ii)		$f''(x) = 36x^{-4}$	M1 A1 [2]	Attempt to differentiate their (i) i.e. at least one term "correct" Fully correct cao No follow through for A mark	Allow constant differentiated to zero ISW incorrect simplification after correct expression

6.

(i)	$y = 5x^{-2} - \frac{1}{4}x^{-1} + x$ $\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$	M1 A1 A1 A1	4	x^{-2} used for $\frac{1}{x^2}$ OR x^{-1} used for $\frac{1}{x}$ soi, **OR** x correctly differentiated kx^{-3} or kx^{-2} from differentiating Two fully correct terms Completely correct	Look out for: $y = 5x^{-2} - 4x^{-1} + x$ followed by $\frac{dy}{dx} = -10x^{-3} + 4x^{-2} + 1$ and then the correct answer. This is **M1 A1 A1 A0** $4x^{-1}$ is **NOT** a misread
(ii)	$\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$	M1 A1	 2 6	Attempt to differentiate their $\frac{dy}{dx}$ (one term correctly differentiated) Completely correct	Allow a sign slip in coefficient for M mark NB Only penalise "+ c" first time seen in the question

7.

Considers $\frac{3(x+h)^2 - 3x^2}{h}$	B1	2.1
Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
So gradient $= \frac{6xh + 3h^2}{h} = 6x + 3h$ or $\frac{6x\delta x + 3(\delta x)^2}{\delta x} = 6x + 3\delta x$	A1	1.1b
States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ *	A1*	2.5
		(4 marks)

8.

(a)	$(x^2+4)(x-3)=x^3-3x^2+4x-12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\dfrac{x^3-3x^2+4x-12}{2x}=\dfrac{x^2}{2}-\dfrac{3}{2}x+2-6x^{-1}$	M1: Attempt to divide each term by $2x$. The powers of x of at least two terms must follow from their expansion. Allow an attempt to multiply by $2x^{-1}$ A1: Correct expression. May be un-simplified but powers of x must be combined e.g. $\dfrac{x^2}{2}$ not $\dfrac{x^3}{2x}$	M1A1
	$\dfrac{dy}{dx}=x-\dfrac{3}{2}+\dfrac{6}{x^2}$ oe e.g. $\dfrac{2x^3-3x^2+12}{2x^2}$	ddM1: $x^n \to x^{n-1}$ or $2\to 0$ **Dependent on both previous method marks.** A1: $x-\dfrac{3}{2}+\dfrac{6}{x^2}$ oe and isw Accept $1x$ or even $1x^1$ but not $\dfrac{2x}{2}$ and not x^0. If they lose the previous A1 because of an **incorrect constant only** then allow recovery here and in part (b) for a correct derivative.	ddM1A1
(b)	At $x=-1,\ y=10$	Correct value for y	B1
	$\left(\dfrac{dy}{dx}=\right)-1-\dfrac{3}{2}+\dfrac{6}{1}=3.5$	M1: Substitutes $x=-1$ into their expression for dy/dx A1: 3.5 oe cso	M1A1
	$y-'10'='3.5'(x--1)$	Uses their **tangent gradient which must come from calculus** with $x=-1$ and their numerical y with a correct straight line method. If using $y=mx+c$, this mark is awarded for correctly establishing a value for c.	M1
	$2y-7x-27=0$	$\pm k(2y-7x-27)=0$ cso	A1
			(5)
			(10 marks)

9.

$P(4, -1)$ lies on C where $f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3,\ x > 0$	
$f'(4) = \frac{1}{2}(4) - \frac{6}{\sqrt{4}} + 3\ ;\ = 2$	M1; A1
T: $y - -1 = 2(x - 4)$	dM1
T: $y = 2x - 9$	A1
	[4]

10.

$[\frac{dy}{dx} =]\ 32x^3$ c.a.o.	**M1**		
substitution of $x = ½$ in their $\frac{dy}{dx}$	**M1**	[= 4]	must see kx^3
grad normal = $\frac{-1}{their 4}$	**M1**		their 4 must be obtained by calculus
when $x = ½$, $y = 4\ ½$ o.e.	**B1**		
$y - 4\frac{1}{2} = -\frac{1}{4}(x - \frac{1}{2})$ i.s.w	**A1**	$y = -\frac{1}{4}x + 4\frac{5}{8}$ o.e.	

11.

$\frac{dy}{dx} = -12x^{-3}$	M1 A1	Attempt to differentiate (i.e. kx^{-3} seen) Correct derivative	"+ C" is A0
When $x = 2$, $\frac{dy}{dx} = -\frac{3}{2}$	A1	Correct value of $\frac{dy}{dx}$. Allow equivalent fractions.	
Gradient of normal = $\frac{2}{3}$	B1 FT	Follow through their evaluated $\frac{dy}{dx}$	Must be processed correctly
When $x = 2$, $y = -\frac{7}{2}$	B1	Correct y coordinate, accept equivalent forms	
$y + \frac{7}{2} = \frac{2}{3}(x - 2)$	M1	Correct equation of straight line through (2, their evaluated y), any non-zero gradient	
$4x - 6y - 29 = 0$	A1 [7]	Correct equation in required form i.e. $k(4x - 6y - 29) = 0$ for integer k. Must have "=0".	

12.

Solution	Mark	Total	Comment
Gradient of the line $3y-2x=1$ is $\frac{2}{3}$	B1		(Gradient) $\frac{2}{3}$ seen or used. Condone 0.66, 0.67 or better for $\frac{2}{3}$.
$\frac{dy}{dx}=\frac{1}{2}x^{-0.5}$	B1		Correct differentiation of $x^{\frac{1}{2}}$
At A, $\frac{1}{2}x^{-0.5}=\frac{2}{3}$	M1		c's $\frac{dy}{dx}$ expression = c's numerical gradient of given line.
$A\left(\frac{9}{16},\frac{3}{4}\right)$	A1		Correct exact coordinates of A
Eqn of tang at A: $y-\frac{3}{4}=\frac{2}{3}\left(x-\frac{9}{16}\right)$	A1	5	ACF eg $y=\frac{2}{3}x+\frac{3}{8}$ or eg $3y-2x=\frac{9}{8}$ must be exact
Total		**5**	

13.

Solution	Mark	Total	Comment
$\frac{1}{x^2} = x^{-2}$	B1		$\frac{1}{x^2} = x^{-2}$. PI by its **correct** derivative
$(y = \frac{1}{x^2} + 4x)$ $(\frac{dy}{dx} =)$ $-2x^{-3} + 4$	M1		Correct differentiation of either $\frac{1}{x^2}$ or $4x$
	A1	3	Correct $\frac{dy}{dx}$ ACF
When $x = -1$, $\frac{dy}{dx} = -2(-1)^{-3} + 4$ $(= 6)$	M1		Attempt to find the value of $\frac{dy}{dx}$ when $x = -1$
Gradient of normal $= -\frac{1}{6}$	m1		Correct use of $m \times m' = -1$, with c's value of $\frac{dy}{dx}$ when $x = -1$
(Eqn of normal) $y + 3 = -\frac{1}{6}(x + 1)$	A1F	3	A correct ft equation for normal with signs simplified; ft on c's $\frac{dy}{dx}$ expression in **(a)** **SC** $\frac{dy}{dx}$ = const in **(a)**, mark **(b)** as M1A1F eg for $\frac{dy}{dx}$=4 in (a); grad of normal= $-\frac{1}{4}$ (M1) , eqn $y + 3 = -\frac{1}{4}(x + 1)$ (A1F)
$-2x^{-3} + 4 = -12$	M1		C's answer to **(a)** equated to -12 (or to 12) seen or used.
$x^{-3} = 8$	A1F		PI Correct rearrangement of $ax^{-n} + b = \pm 12$ or $\frac{a}{x^n} + b = \pm 12$ OE to form $x^{-n} = q$ or to form $x^n = p$, but only ft in case of n positive
$x = 0.5$	A1		$x = 0.5$ OE
When $x = 0.5$, $y = 6$	A1F		Correct ft y coordinate from $y_c = x_c^{-2} + 4x_c$. Only ft if values are exact.
(Eqn of tangent) $y - 6 = -12(x - 0.5)$ (or eg $y = -12x + 12$)	A1	5	Correct tangent equation ACF Apply ISW after ACF
Total		**11**	

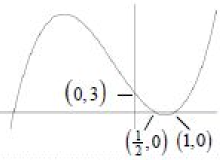
14.

(i)		$y' = 3x^2 - 5$	M1	
		their $y' = 0$	M1	
		$(1.3,\ -4.3)$ cao	A1	or A1 for $x = \pm\sqrt{\frac{5}{3}}$ oe soi
		$(-1.3,\ 4.3)$ cao	A1	allow if not written as co-ordinates if pairing is clear
			[4]	
(ii)		crosses axes at (0, 0)	B1	condone x and y intercepts not written as co-ordinates; may be on graph
		and $(\pm\sqrt{5}\ , 0)$	B1	±(2.23 to 2.24) implies $\pm\sqrt{5}$
		sketch of cubic with turning points in correct quadrants and of correct orientation and passing through origin	B1	
		x-intercepts $\pm\sqrt{5}$ marked	B1	may be in decimal form (±2.2…)
			[4]	
(iii)		substitution of $x = 1$ in $f'(x) = 3x^2 - 5$	M1	
		-2	A1	
		$y - -4 = (\text{their } f'(1)) \times (x - 1)$ oe	M1*	or $-4 = -2\times(1) + c$
		$-2x - 2 = x^3 - 5x$ and completion to given result www	M1dep*	
		use of Factor theorem in $x^3 - 3x + 2$ with -1 or ± 2	M1	or any other valid method; must be shown
		$x = -2$ obtained correctly	A1	
			[6]	

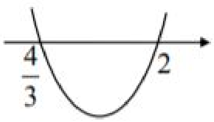
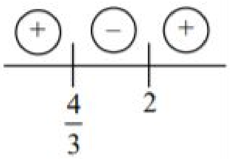
15.

$\frac{dy}{dx} = x^2 - 9x^{-2}$	B1	x^2 from differentiating first term	
	M1	kx^{-2}	
	A1	$-9x^{-2}$ (no $+ c$)	
Gradient of line = 8	B1		
$x^2 - 9x^{-2} = 8$	M1	Equate their $\frac{dy}{dx}$ to 8 (or their gradient of line, if clear)	Note: If equated to +/-1/8 then M0 but the next M1 and its dependencies are available
$x^4 - 8x^2 - 9 = 0$ $k^2 - 8k - 9 = 0$	*M1	Use a correct substitution to obtain a 3 term quadratic or factorise into 2 brackets each containing x^2	If no substitution stated and treated as a quadratic (e.g. quadratic formula), no more marks until square rooting seen
$(k-9)(k+1) = 0$	DM1	Correct method to solve 3 term quadratic – **dependent on previous M1**	SC: If spotted after first five marks- (3, 12) **B1** (-3, -12) **B1** Justifies exactly two solutions **B3**
$k = 9$ (don't need $k = -1$)	A1	No extras	
$x = 3,\ -3$	DM1	Attempt to find x by square rooting – accept one value	
$y = 12,\ -12$	A1 [10]	No extras	

16.

(i)		B1 B1 B1	3	+ve cubic with 3 distinct roots (0, 3) labelled or indicated on y-axis (-3, 0), ($\frac{1}{2}$, 0) and (1, 0) labelled or indicated on x-axis and no other x- intercepts	For first **B1**, left end of curve must finish below x axis and right end must end above x axis. Allow slight wrong curvature at one end but not both ends. No cusp at either turning point. No straight lines drawn with a ruler. Condone (0, 3) as maximum point. To gain second and third **B** marks, there must be an attempt at a curve, not just points on axes. Final **B1** can be awarded for a negative cubic.
(ii)	$2x^2 + 5x - 3,\ x^2 + 2x - 3,\ 2x^2 - 3x + 1$ $(2x^2 + 5x - 3)(x - 1)$ $2x^3 + 3x^2 - 8x + 3$ $\frac{dy}{dx} = 6x^2 + 6x - 8$ When $x = 1$, gradient = 4	B1 M1 A1 M1 A1 A1	6	Obtain one quadratic factor (can be unsimplified) Attempt to multiply a quadratic by a linear factor Attempt to differentiate (one non-zero term correct) Fully correct expression www Confirms gradient = 4 at x = 1 **AG	<u>Alternative for first 3 marks:</u> Attempt to expand all 3 brackets with an appropriate number of terms (including an x^3 term) **M1** Expansion with at most 1 incorrect term **A1** Correct, answer (can be unsimplified) **A1** Allow if done in part(i) please check.
(iii)	Gradient of $l = 4$ On curve, when $x = -2,\ y = 15$ $y - 15 = 4(x + 2)$ $y = 4x + 23$	B1 B1 M1 A1	4	May be embedded in equation of line Correct y coordinate Correct equation of line using their values Correct answer **in correct form**	M mark is for any equation of line with any non-zero numerical gradient through (-2, their evaluated y)
(iv)	Attempt to find gradient of curve when $x = -2$ $6(-2)^2 + 6(-2) - 8 = 4$ So line is a tangent	M1 A1 A1	3 16	Substitute $x = -2$ into their $\frac{dy}{dx}$ Obtain gradient of 4 CWO Correct conclusion	<u>Alternatives</u> 1) Equates equation of l to equation of curve and attempts to divide resulting cubic by $(x + 2)$ **M1** Obtains $(x + 2)^2(2x - 5)$ (=0) **A1** Concludes repeated root implies tangent at x = -2 **A1** 2) Equates their gradient function to 4 and uses correct method to solve the resulting quadratic **M1** Obtains $(x + 2)(x - 1) = 0$ oe **A1** Correctly concludes gradient = 4 when $x = -2$ **A1**

17.

(a)(i)	$\left.\begin{array}{l}(\text{Increasing} \Rightarrow) \frac{dy}{dx} > 0 \\ 20x - 6x^2 - 16 > 0\end{array}\right\}$ either $\Rightarrow 6x^2 - 20x + 16 < 0$ *or* (2) $(10x - 3x^2 - 8) > 0$ $\Rightarrow 3x^2 - 10x + 8 < 0$	M1 A1	 2	correct interpretation of y increasing must see at least one of these steps before final answer for A1 CSO **AG** no errors in working
(ii)	$(3x-4)(x-2)$ CVs are $\frac{4}{3}$ and 2 $\frac{4}{3} < x < 2$ ***Mark their final line as their answer***	M1 A1 M1 A1	 4	correct factors or correct use of quadratic equation formula as far as $\frac{10 \pm \sqrt{4}}{6}$ condone $\frac{8}{6}$ and $\frac{12}{6}$ here but not in final line sketch or sign diagram or $2 > x > \frac{4}{3}$ accept $x < 2$ **AND** $x > \frac{4}{3}$ but **not** $x < 2$ **OR** $x > \frac{4}{3}$ **nor** $x < 2$, $x > \frac{4}{3}$

(b)(i)	$x = 2$; $\left(\frac{dy}{dx} =\right)$ $40 - 24 - 16$	M1		sub $x = 2$ into $\frac{dy}{dx}$ and simplify terms
	$\frac{dy}{dx} = 0 \Rightarrow$ tangent at P is parallel to the x-axis	A1	2	must be all correct working plus statement
(ii)	$x = 3$; $\frac{dy}{dx} = 20\times3 - 6\times3^2 - 16$	M1		must attempt to sub $x = 3$ into $\frac{dy}{dx}$
	$(= 60–54–16) \quad = -10$	A1		
	Gradient of normal $= \frac{1}{10}$	A1√		$\frac{-1}{\text{"their } -10\text{"}}$
	Normal: $(y - -1) = \text{'their grad'}(x - 3)$	m1		normal attempted with correct coordinates used and gradient obtained from their $\frac{dy}{dx}$ value
	$y + 1 = \frac{1}{10}(x - 3)$	A1		any correct form, eg $10y = x - 13$ but must simplify $- -$ to $+$
	(Equation of tangent at P is) $y = 3$	B1		
	$x = 43$	A1	7	CSO; $\Rightarrow R(43, 3)$
	Total		**15**	

18.

1.(a)	Substitutes $x = 2$ into $y = 20 - 4\times2 - \frac{18}{2}$ and gets 3	B1
	$\frac{dy}{dx} = -4 + \frac{18}{x^2}$	M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	Method 1: States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3) / Method 2: Or: Check that (2, 3) lies on the line $y = -2x + 7$	dM1
	Method 1: to deduce that $y = -2x + 7$ * / Method 2: Deduce equation of normal as it has the same gradient and passes through a common point	A1*
		(6)
(b)	Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$	M1 A1
	Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$	
	$(2x - 9)(x - 2) = 0$ so $x =$ or $(y - 3)(y + 2) = 0$ so $y =$	dM1
	$x = \frac{9}{2}, y = -2$	A1, A1
		(5)
		(11 marks)

19.

5(a)		B1		For either 6 or $6x^0$
	$\frac{dy}{dx} = 6 - 3x^{\frac{1}{2}}$	M1		$Ax^{\frac{3}{2}-1}$, $A \neq 0$ OE
		A1	3	$6 - 3x^{\frac{1}{2}}$ or $6 - 3\sqrt{x}$ with no '+c' [If unsimplified here, A1 can be awarded retrospectively if correct simplified expression is seen explicitly in (b)(i).]
(b)(i)	$6 - 3x^{\frac{1}{2}} = 0$	M1		Equating c's $\frac{dy}{dx}$ to 0 PI by correct ft rearrangement of c's dy/dx=0
	$x^{\frac{1}{2}} = 2 \Rightarrow x = 2^2$	m1		$x^{\frac{1}{2}} = k$ ($k>0$), to $x = k^2$. PI by correct value of x if no error seen
	$M(4, 8)$	A1	3	SC If M0 award B1 for (4, 8)
(ii)	Eqn of normal at M is $x = 4$	B1F	1	Ft on x = c's x_M
(c)(i)	When $x = \frac{9}{4}$, $\frac{dy}{dx} = 6 - 3\times\frac{3}{2} = \frac{3}{2}$	M1		Attempt to find $\frac{dy}{dx}$ when $x = \frac{9}{4}$
	Gradient of normal at $P = -\frac{2}{3}$	m1		$m \times m' = -1$ used
	Eqn of normal: $y - \frac{27}{4} = -\frac{2}{3}\left(x - \frac{9}{4}\right)$	A1		ACF eg $y = -\frac{2}{3}x + \frac{33}{4}$
	$12y - 81 = -8x + 18 \Rightarrow 8x + 12y = 99$	A1	4	Coeffs and constant must now be positive integers, but accept different order eg $12y + 8x = 99$
(ii)	$8(4) + 12y = 99$	M1		Solving c's answer (b)(ii), (must be in form x = positive const), with c's answer (c)(i). PI by correct earlier work and correct coordinates for R.
	$R\left(4, \frac{67}{12}\right)$	A1	2	Accept 5.58 or better as equivalent to $\frac{67}{12}$
	Total		**13**	

20.

(i)	$\frac{dy}{dx} = 6 - 2x$	M1 A1		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x = 5$, $6 - 2x = -4$	M1		Substitute $x = 5$ into their $\frac{dy}{dx}$	
	When $x = 5$, $y = 12$	B1		Correct y coordinate	
	$y - 12 = -4(x - 5)$	M1		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient
	$4x + y - 32 = 0$	A1	6	Shows rearrangement to correct form	If using $y = mx + c$ must attempt at evaluating c Allow any correct form e.g. $0 = 2y + 8x - 64$ etc.
(ii)	Q is point (8, 0)	B1ft		ft from line in (i)	
	Midpoint of $PQ = \left(\frac{5+8}{2}, \frac{12+0}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$= \left(\frac{13}{2}, 6\right)$	A1	3		Do not accept $\left(\frac{13}{2}, \frac{12}{2}\right)$
(iii)	$6 - 2x = 0$	M1		Solution of their $\frac{dy}{dx} = 0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x = -\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x = 3$	A1	2	Allow from $\pm[16 - (x-3)^2]$, $\pm[6 - 2x = 0]$	
(iv)	$x < 3$	M1		$x <$ their3 or $x >$ their3 **OR** attempt to solve their $\frac{dy}{dx} > 0$	May solve $\frac{dy}{dx} = 0$ then use $\frac{d^2y}{dx^2} < 0$ implies maximum point for the method mark, or sketch of curve
		A1	2 13	Allow from $\pm[16 - (x-3)^2]$, $\pm[6 - 2x = 0]$ in (iii)	Allow $x \le 3$

21.

(i)	$\frac{dy}{dx} = 6 - 2x$	M1 A1		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x = 5$, $6 - 2x = -4$	M1		Substitute $x = 5$ into their $\frac{dy}{dx}$	
	When $x = 5$, $y = 12$	B1		Correct y coordinate	
	$y - 12 = -4(x - 5)$	M1		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient
	$4x + y - 32 = 0$	A1	6	Shows rearrangement to correct form	If using $y = mx + c$ must attempt at evaluating c Allow any correct form e.g. $0 = 2y + 8x - 64$ etc.
(ii)	Q is point (8, 0)	B1ft		ft from line in (i)	
	Midpoint of $PQ = \left(\frac{5+8}{2}, \frac{12+0}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$= \left(\frac{13}{2}, 6\right)$	A1	3		Do not accept $\left(\frac{13}{2}, \frac{12}{2}\right)$
(iii)	$6 - 2x = 0$	M1		Solution of their $\frac{dy}{dx} = 0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x = -\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x = 3$	A1	2	Allow from $\pm[16 - (x-3)^2]$, $\pm[6 - 2x = 0]$	
(iv)	$x < 3$	M1		$x <$ their3 or $x >$ their3 **OR** attempt to solve their $\frac{dy}{dx} > 0$	May solve $\frac{dy}{dx} = 0$ then use $\frac{d^2y}{dx^2} < 0$ implies maximum point for the method mark, or sketch of curve
		A1	2 13	Allow from $\pm[16 - (x-3)^2]$, $\pm[6 - 2x = 0]$ in (iii)	Allow $x \le 3$

22.

3(a)	$\left(\frac{dV}{dt}=\right)\frac{3t^2}{4}-3$	M1 A1	2	one of these terms correct all correct (no + *c* etc)
(b)(i)	$t=1\Rightarrow\frac{dV}{dt}=\frac{3}{4}-3$ $=-2\frac{1}{4}$	M1 A1cso	2	substituting $t=1$ into their $\frac{dV}{dt}$ (–2.25 OE) *BUT* must have $\frac{dV}{dt}$ correct
(ii)	Volume is decreasing when $t=1$ because $\frac{dV}{dt}<0$	E1√	1	must have used $\frac{dV}{dt}$ in (b)(i) or starts again must state that $\frac{dV}{dt}<0$ (or $-2\frac{1}{4}<0$ etc) ft increasing plus explanation if their $\frac{dV}{dt}>0$
(c)(i)	$\left(\frac{dV}{dt}=0\Rightarrow\right)\frac{3t^2}{4}-3=0$ $\Rightarrow t^2=4$ $t=2$	M1 A1√ A1cso	3	PI by "correct" equation being solved obtaining $t^n=k$ correctly from their $\frac{dV}{dt}$ withhold if answer left as $t=\pm 2$
(ii)	$\left(\frac{d^2V}{dt^2}=\right)\frac{3t}{2}$ When $t=2$, $\frac{d^2V}{dt^2}=3$ or $\frac{d^2V}{dt^2}>0$ $\Rightarrow$ minimum	B1√ M1 A1cso	3	(condone unsimplified) ft their $\frac{dV}{dt}$ ft their $\frac{d^2V}{dt^2}$ and value of *t* from (c)(i)
	Total		**11**	

23.

(a)(i)	$\left(\frac{dy}{dx}=\right)\ 5x^4-6x+1$	M1 A1 A1	3	one term correct another term correct all correct (no $+c$ etc)
(ii)	$\left(\frac{d^2y}{dx^2}=\right)\ 20x^3-6$	B1√	1	FT 'their' $\frac{dy}{dx}$
(b)	$x=-1\Rightarrow\frac{dy}{dx}=5(-1)^4-6(-1)+1\quad(=12)$	M1		must sub $x=-1$ into 'their' $\frac{dy}{dx}$
	$\Rightarrow\ y=12(x+1)$	A1cso	2	any correct form with $(x--1)$ simplified condone $y=12x+c,\quad c=12$
(c)	$x=1\Rightarrow\frac{dy}{dx}=5-6+1$	M1		sub $x=1$ into their $\frac{dy}{dx}$
	$\frac{dy}{dx}=0\Rightarrow$ stationary point	A1cso		shown $=0$ plus correct statement
	when $x=1,\ \frac{d^2y}{dx^2}=14$			or $\frac{d^2y}{dx^2}=20-6>0$
	$\Rightarrow$ (B is a) minimum (point)	E1	3	$\Rightarrow$ (B is a) minimum (point) must have correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for E1

24.

$6x^2+18x-24$	B1		
their $6x^2+18x-24=0$ or >0 or ≥ 0	M1		or sketch of $y=6x^2+18x-24$ with attempt to find x-intercepts
-4 and $+1$ identified oe	A1		
$x<-4$ and $x>1$ cao	A1	or $x\leq-4$ and $x\geq 1$	if **B0M0** then **SC2** for fully correct answer
	[4]		

25.

6(a)	$\sqrt{x} = x^{0.5}$	B1		$\sqrt{x} = x^{0.5}$ or $\sqrt{x} = x^{\frac{1}{2}}$ seen or used
	$\frac{12 + x^2\sqrt{x}}{x} = \frac{12 + x^{2.5}}{x}$ $= 12x^{-1} + x^{1.5}$	B1		$12x^{-1}$ or $p = -1$
		B1	3	$x^{1.5}$ or $q = \frac{3}{2}$ (=1.5)
(b)(i)	$\frac{dy}{dx} = -12x^{-2}$	B1F		Ft on c's p only if c's p is a negative integer
	$+1.5x^{0.5}$	B1F	2	Ft on c's q only if c's q is a pos non-integer
(ii)	When $x = 4,\ y = 11$	B1		
	When $x = 4,\ \frac{dy}{dx} = \frac{-12}{16} + 3 = \frac{9}{4}$	M1		Attempt to find $\frac{dy}{dx}$ when $x = 4$ PI
	Gradient of normal $= -\frac{4}{9}$	m1		$m \times m' = -1$ used
	Eqn of normal: $y - 11 = -\frac{4}{9}(x - 4)$	A1	4	ACF eg $4x + 9y = 115$
(iii)	At St Pt $\frac{dy}{dx} = -12x^{-2} + 1.5x^{0.5} = 0$	M1		Equating c's $\frac{dy}{dx}$ to zero.
	$\Rightarrow x^2x^{0.5} = 8,\ \Rightarrow\ x^{\frac{5}{2}} = 8\ \Rightarrow\ x = 8^{\frac{2}{5}}$	A1		A correct eqn in the form $x^n = c$ or $x = c^{\frac{1}{n}}$ correctly obtained.
	$\Rightarrow\ x = (2^3)^{\frac{2}{5}}\ \Rightarrow\ x = 2^{\frac{6}{5}}$	A1	3	CSO $x = 2^{\frac{6}{5}}$. All working must be correct and in an exact form. If 'x=0' also appears then A0 CSO
			12	

26.

(i)	$\frac{dy}{dx} = 6x + 6x^{-2}$	M1 A1		Attempt to differentiate (one non-zero term correct) Completely correct	NB – $x = -1$ (and therefore possibly $y = 7$) can be found from equating the incorrect differential
	$6x + \frac{6}{x^2} = 0$	M1		Sets their $\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 6x + 6$ to 0. This could score M1A0 M1A0A1 ft
	$x = -1$	A1		Correct value for x - www	
	$y = 7$	A1 ft	5	Correct value of y for *their* value of x	If more than one value of x found, allow A1 ft for one correct value of y
(ii)	$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$	M1		Correct method e.g. substitutes their x from (i) into their $\frac{d^2y}{dx^2}$ (must involve x) and considers sign.	Allow comparing signs of their $\frac{dy}{dx}$ either side of their "– 1", comparing values of y to their "7"
	When $x = -1$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	A1 ft	2 7	ft from their $\frac{dy}{dx}$ differentiated correctly and correct substitution of *their* value of x and consistent final conclusion NB If second derivate evaluated, it must be correct (18 for $x = -1$). If more than one value of x used, max M1 A0	SC $\frac{d^2y}{dx^2}$ = a constant correctly obtained from their $\frac{dy}{dx}$ and correct conclusion (ft) B1

27.

(i)	$\frac{dy}{dx} = 4x^3 + 32$ $4x^3 + 32 = 0$ $x = -2$ $y = -48$	M1 A1 M1 A1 A1 FT [5]	Attempt to differentiate (one term correct) Completely correct Sets their $\frac{dy}{dx} = 0$ (can be implied) Correct value for x (**not** ± 2) www Correct value of y for *their* single non-zero value of x	"+ C" is A0 e.g. (2, 80), (4, 384), (− 4, 128), (8, 4352), (− 8, 3840)
(ii)	$\frac{d^2y}{dx^2} = 12x^2$ When $x = -2$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	M1 A1 [2]	Correct method for determining nature of a stationary point – see right hand column Fully correct for $x = -2$ only	e.g. evaluating second derivate at x = "−2" and stating a conclusion Evaluating $\frac{dy}{dx}$ either side of x = "−2" Evaluating y either side of x = "−2"
(iii)	$x > -2$	B1 FT [1]	ft from single x value in (i) consistent with (ii)	**Do not accept** $x \geq -2$

28.

(a)	$\left\{\frac{dy}{dx} =\right\}$ $2x - 16x^{-\frac{1}{2}}$	M1 A1
	$2x - 16x^{-\frac{1}{2}} = 0 \Rightarrow x^{\frac{3}{2}} =, x^{-\frac{3}{2}} =$,or $2x - =16x^{-\frac{1}{2}}$ then squared then obtain $x^3 =$ [or $2x - 16x^{-\frac{1}{2}} = 0 \Rightarrow x = 4$ (no wrong work seen)]	M1
	$(x^{\frac{3}{2}} = 8 \Rightarrow) x = 4$	A1
	$x = 4,\ y = 4^2 - 32\sqrt{4} + 20 = -28$ (ignore $y = 100$ as second answer)	M1 A1
		(6)
b)	$\left\{\frac{d^2y}{dx^2} =\right\}$ $2 + 8x^{-\frac{3}{2}}$	M1 A1
	$(\frac{d^2y}{dx^2} > 0 \Rightarrow) y$ is a minimum (there should be no wrong reasoning)	A1
		(3) **[9]**

29.

(a)	$\frac{dy}{dx} = -3 + \frac{12}{x^4}$ *or* $-3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ ($x^1 \to x^0$ or $x^{-3} \to x^{-4}$ or $6 \to 0$) A1: Correct derivative	M1 A1
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = ...$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\frac{dy}{dx} = -3 + \frac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]

30.

1(a)	$\frac{dy}{dx} = 18 + 6x - 12x^2$	M1 A1 A1	3	one of these terms correct another term correct all correct (no + *c* etc) (penalise + *c* once only in question)
(b)	$18 + 6x - 12x^2 = 0$	M1		putting their $\frac{dy}{dx} = 0$, PI by attempt to solve or factorise
	$6\ (3-2x)(x+1) \quad (=0)$	m1		attempt at factors of **their quadratic** or use of quadratic equation formula
	$x = -1,\ x = \frac{3}{2}$ OE	A1	3	must see both values unless $x = -1$ is verified separately If M1 not scored, award SC B1 for verifying that $x = -1$ leads to $\frac{dy}{dx} = 0$ and a further SC B2 for finding $x = \frac{3}{2}$ as other value
(c)(i)	$\frac{d^2y}{dx^2} = 6 - 24x$	B1√		FT their $\frac{dy}{dx}$ but $\frac{d^2y}{dx^2}$ must be correct if 3 marks earned in part (a)
	When $x = -1$, $\frac{d^2y}{dx^2} = 6 - (24 \times -1)$	M1		Sub $x = -1$ into 'their' $\frac{d^2y}{dx^2}$
	$\frac{d^2y}{dx^2} = 30$	A1cso	3	
(ii)	Minimum point	E1√	1	must have a value in (c)(i) FT "maximum" if their value of $\frac{d^2y}{dx^2} < 0$
	Total		**10**	

31.

(i)	$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1 A1	Attempt to differentiate, at least two non-zero terms correct Fully correct	
	When $x = 4$, $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$	These Ms may be awarded in either order
	$\frac{dy}{dx} = 0$ gives $a = 13$	M1 A1 **[5]**	Sets their $\frac{dy}{dx}$ to 0. Must be seen	
(ii)	$\frac{d^2y}{dx^2} = 12x - 26$	M1	Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign	**Alternate valid methods include:** 1) Evaluating gradient at either side of 4($x > \frac{1}{3}$) e.g. at 3, –16 at 5, 28 2) Evaluating $y = -46$ at 4 and either side of 4 ($x > \frac{1}{3}$) e.g. (3, – 37), (5, – 33)
	When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum	A1 **[2]**	www	If using alternatives, working must be fully correct to obtain the **A** mark
(iii)	$6x^2 - 26x + 8 = 0$ $(3x - 1)(x - 4) = 0$ $x = \frac{1}{3}$	M1 M1 A1 **[3]**	Sets their derivative to zero Correct method to solve quadratic **(appx 1)** oe	Could be $(6x - 2)(x - 4) = 0$ or $(3x - 1)(2x - 8) = 0$

32.

2(a)	$\left(\frac{dy}{dt} =\right) \frac{4t^3}{8} - 2t$	M1 A1	2	one of these terms correct all correct (no + *c* etc)
(b)(i)	$t = 1 \Rightarrow \frac{dy}{dt} = \frac{4}{8} - 2$	M1		Correctly sub $t = 1$ into their $\frac{dy}{dt}$
	$= -1\frac{1}{2}$	A1cso	2	must have $\frac{dy}{dt}$ correct (watch for t^3 etc)
(ii)	$\frac{dy}{dt} < 0$			must have used $\frac{dy}{dt}$ in part (b)(i)
	$\Rightarrow$ (height is) **decreasing** (when $t = 1$)	E1√	1	must state that " $\frac{dy}{dt} < 0$ " or "$-1.5 < 0$" or the equivalent in words FT their value of $\frac{dy}{dt}$ with appropriate explanation and conclusion
(c)(i)	$\left(\frac{d^2y}{dt^2} =\right) \frac{4}{8} \times 3t^2 - 2$	M1		Correctly differentiating their $\frac{dy}{dt}$ even if wrongly simplified
	$\left(t = 2, \frac{d^2y}{dt^2} =\right)$ 4	A1cso	2	Both derivatives correct and simplified to 4
(ii)	$\Rightarrow$ minimum	E1√	1	FT their numerical value of $\frac{d^2y}{dt^2}$ from part (c) (i)
	Total		**8**	

33.

		Mark		
(i)	$y = -x^3 - 3x^2 + 4x - kx + k$	M1 A1	Attempt to multiply out brackets Can be unsimplified	Must have $\pm x^3$ and 5 or 6 terms
	$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$	M1 A1	Attempt to differentiate **their** expansion (**M0** if signs have changed throughout)	**If using product rule:** Clear attempt at correct rule **M1*** Differentiates both parts correctly **A1** Expand brackets of both parts ***DM1**
	When $x = -3$, $\frac{dy}{dx} = 0$	M1* DM1*	Sets $\frac{dy}{dx} = 0$ Substitutes $x = -3$ into their $\frac{dy}{dx} = 0$	
	$-27 + 18 + 4 - k = 0$ $k = -5$	A1 **[7]**	**www**	Then as main scheme
(ii)	$\frac{d^2y}{dx^2} = -6x - 6$	M1	Evaluates second derivative at $x = -3$ or other fully correct method	**Alternate valid methods include:** 1) Evaluating gradient at either side of -3 2) Evaluating y at either side of -3 3) Finding other turning point and stating "negative cubic so min before max"
	When $x = -3$, $\frac{d^2y}{dx^2}$ is positive so min point	A1 **[2]**	No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in k value)	
(iii)	$-3x^2 - 6x + 9 = 9$	M1	Sets their gradient function from (i) (or from a restart) to 9	Allow first **M** even if k not found but look out for correct answer from wrong working.
	$3x(x+2) = 0$ $x = 0$ or $x = -2$	A1	Correct x-values	<u>**SEE NEXT PAGE FOR ALTERNATIVE METHODS**</u>
	When $x = 0$, $y = -9$ for line $y = -5$ for curve	M1	One of their x-values substituted into both curve and line/substituted into one and verified to be on the other	**Note:** Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent
	When $x = -2$, $y = -27$ for line $y = -27$ for curve	M1	Conclusion that $x = -2$ is the correct value **<u>or</u>** Second x-value substituted into both curve and line/verified as above	<u>If curve equated to line before differentiating:</u>
	$x = -2, y = -27$	A1 **[5]**	$x = -2, y = -27$ **www** ***(Check k correct)***	**M0 A0**, can get **M1M1** but **A0 ww** Maximum mark **2/5**

34.

		Mark		
(i)	$x^3 - 3x^2 + 5x + 2x^2 - 6x + 10$ $= x^3 - x^2 - x + 10$	M1	Attempt to multiply out brackets	<u>Alternative for product rule</u> Attempt to use product rule **M1** Expand brackets of both parts **M1** Then as main scheme
	$\frac{dy}{dx} = 3x^2 - 2x - 1$	M1 M1*	Attempt to differentiate their cubic Sets their $\frac{dy}{dx} = 0$	
	$(3x + 1)(x - 1) = 0$	M1	Correct method to solve quadratic	
	$x = -\frac{1}{3}$ or $x = 1$	A1	Correct x values of turning points found **www**	Any extra values for turning points loses all three **A** marks
	$\frac{d^2y}{dx^2} = 6x - 2$, $x = 1$ gives +ve (4) Min point at $x = 1$	M1dep A1	Valid method to establish which is min point with a conclusion Correct conclusion for $x = 1$ found from correct factorisation (even if other root incorrect)	(eg by sketching positive cubic, second diff method for either of their x values, y co-ords etc.)
	$y = 9$ found	A1	**www** for (1, 9) given as minimum point (ignore other point here)	If constant incorrect in initial expansion, max **5/8**
(ii)	$(-3)^2 - 4 \times 1 \times 5$ $= -11$	M1 A1 **[2]**	Uses $b^2 - 4ac$	$\sqrt{b^2 - 4ac}$ is **M0**
(iii)		B2 **[2]**	Fully correct argument - no extra incorrect statements e.g. 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ and stating it's a positive cubic 2) Sketch of positive cubic with one root at (–2, 0) and a min point at (1, 9) (f/t positive $y(1)$ from (i))	Award **B1** for either of: 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ 2) Sketch of positive cubic with one root at (–2, 0) and a min point with y coordinate positive or 0

35.	(a)(i)	$(SA=)\ \pi r^2+2\pi rh$	B1		correct surface area
		$\pi r^2+2\pi rh=48\pi$ $\Rightarrow 2rh=48-r^2 \quad \Rightarrow h=\ldots$	M1		equating *"their" SA* to 48π **and** attempt at $h=$
		$h=\dfrac{48-r^2}{2r}$	A1	3	**or** $h=\dfrac{24}{r}-\dfrac{r}{2}$ **OE**
	(ii)	$V=\pi r^2h=\ldots$ $=\pi f(r)$	M1		correct volume expression & elimination of h using "their" **(a)(i)**
		$V=\pi r^2\left(\dfrac{48-r^2}{2r}\right)=24\pi r-\dfrac{\pi}{2}r^3$	A1	2	**AG (** *be convinced)*
	(b)(i)	$\left(\dfrac{dV}{dr}=\right)\ 24\pi-\dfrac{3}{2}\pi r^2$	M1 A1	2	one term correct all correct, must simplify r^0
	(ii)	$24\pi-\dfrac{3}{2}\pi r^2=0\Rightarrow r^2=\dfrac{48\pi}{3\pi}$	M1		"their" $\dfrac{dV}{dr}=0$ and attempt at $r^n=\ldots$
		$r=4$	A1		from correct $\dfrac{dV}{dr}$
		$\dfrac{d^2V}{dr^2}=-\dfrac{6\pi r}{2}$	B1√		FT "their" $\dfrac{dV}{dr}$
		$\dfrac{d^2V}{dr^2}<0$ when $r=4 \Rightarrow$ Maximum	A1cso	4	explained convincingly, all working and notation correct
		Total		**11**	

36.	(a)(i)	$3x^2 + 3x^2 + xy + xy + 3xy + 3xy$	M1		correct expression for surface area
		$6x^2 + 8xy = 32$			$2(3x^2 + xy + 3xy) = 32$ etc
		$\Rightarrow 3x^2 + 4xy = 16$	A1	2	**AG** be convinced
	(ii)	$(V =) 3x^2y$ OE	M1		correct volume in terms of x and y
		$= 3x\left(\frac{16-3x^2}{4}\right)$ **or** $= 3x^2\left(\frac{16-3x^2}{4x}\right)$			OE
		$= 12x - \frac{9x^3}{4}$	A1	2	CSO **AG** be convinced that all working is correct
	(b)	$\left(\frac{dV}{dx} =\right) 12 - \frac{27}{4}x^2$	M1 A1	 2	one of these terms correct all correct with 9×3 evaluated (no $+ c$ etc)
	(c)(i)	$x = \frac{4}{3} \Rightarrow \frac{dV}{dx} = 12 - \frac{27}{4} \times \left(\frac{4}{3}\right)^2$	M1		attempt to sub $x = \frac{4}{3}$ into '*their*' $\frac{dV}{dx}$
		$\frac{dV}{dx} = 12 - \frac{27}{4} \times \frac{16}{9} = 12 - 12$			or $12 - \frac{432}{36} = 12 - 12$ or $12 - \frac{48}{4} = 0$ etc
		$\frac{dV}{dx} = 0 \Rightarrow$ stationary value	A1	2	CSO; shown = 0 plus statement
	(ii)	$\frac{d^2V}{dx^2} = -\frac{27x}{2}$ OE	B1√		FT for '*their*' $\frac{dV}{dx} = a + bx^2$
		when $x = \frac{4}{3}$, $\frac{d^2V}{dx^2} < 0 \Rightarrow$ maximum	E1√	2	**or** sub of $x = \frac{4}{3}$ into '*their*' $\frac{d^2V}{dx^2}$ $\Rightarrow$ maximum
		(FT "*minimum*" if their $\frac{d^2V}{dx^2} > 0$)			E0 if numerical error seen
		Total		**10**	

37. 8 (a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1	
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1	
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso	(3)
(b)	$P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or ½	M1	
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1	
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1	(3)
(c)	$\left(\dfrac{dP}{dx} =\right) - \dfrac{8}{x^2} + 2$	M1 A1	
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1	
	and so $x = 2$ o.e. (ignore extra answer $x = -2$)	A1	
	$P = 4 + 4 = 8$ (m)	B1	(5)
(d)	$y = \dfrac{4 - \pi}{4}$, (and so width) $= 21$ (cm)	M1, A1	(2) **13**

38. (a)	$2\pi rh + 2\pi r^2 = 800$	B1	
	$h = \dfrac{400 - \pi r^2}{\pi r}$, $V = \pi r^2\left(\dfrac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3$ (*)	M1, M1 A1	(4)
(b)	$\dfrac{dV}{dr} = 400 - 3\pi r^2$	M1 A1	
	$400 - 3\pi r^2 = 0$ $r^2 = ...,$ $r = \sqrt{\dfrac{400}{3\pi}}$ $(= 6.5$ (2 s.f.))	M1 A1	
	$V = 400r - \pi r^3 = 1737 = \dfrac{800}{3}\sqrt{\dfrac{400}{3\pi}}$ (cm^3) (accept awrt 1737 or exact answer)	M1 A1	(6)
(c)	$\dfrac{d^2V}{dr^2} = -6\pi r$, Negative, $\therefore$ maximum (Parts (b) and (c) should be considered together when marking)	M1 A1	(2) [12]