Inspired Learning

1. $f(x) = 2x^3 - 7x^2 + 4x + 4$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

2. **(a)** The polynomial $f(x)$ is given by $f(x) = x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x + 3)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

3. You are given that $f(x) = x^4 - x^3 + x^2 + 9x - 10$.

(i) Show that $x = 1$ is a root of $f(x) = 0$ and hence express $f(x)$ as a product of a linear factor and a cubic factor. **[3]**

(ii) Hence or otherwise find another root of $f(x) = 0$. **[2]**

(iii) Factorise $f(x)$, showing that it has only two linear factors. Show also that $f(x) = 0$ has only two real roots. **[5]**

4. The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + x^2 - 11x + 10$.

(i) Use the factor theorem to find a factor of $f(x)$. **[2]**

(ii) Hence solve the equation $f(x) = 0$, giving each root in an exact form. **[6]**

5. $f(x) = -4x^3 + ax^2 + 9x - 18$, where a is a constant.

Given that $(x - 2)$ is a factor of $f(x)$,

(a) find the value of a,

(2)

(b) factorise $f(x)$ completely,

(3)

(c) find the remainder when $f(x)$ is divided by $(2x - 1)$.

(2)

6. You are given that $f(x) = 2x^3 - 3x^2 - 23x + 12$.

(i) Show that $x = -3$ is a root of $f(x) = 0$ and hence factorise $f(x)$ fully. **[6]**

(ii) Sketch the curve $y = f(x)$. **[3]**

(iii) Find the x-coordinates of the points where the line $y = 4x + 12$ intersects $y = f(x)$. **[4]**

7. The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

8. **(a)** Sketch the curve with equation $y = x^2(x-3)$.

[3 marks]

(b) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^2(x-3) + 20$.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x-4$.

[2 marks]

(ii) Use the Factor Theorem to show that $x+2$ is a factor of $\mathrm{p}(x)$.

[2 marks]

(iii) Express $\mathrm{p}(x)$ in the form $(x+2)(x^2+bx+c)$, where b and c are integers.

[2 marks]

(iv) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root and state its value.

[3 marks]

9. You are given that $\mathrm{f}(x) = x^5 + kx - 20$. When $\mathrm{f}(x)$ is divided by $(x-2)$, the remainder is 18. Find the value of k. **[3]**

10. You are given that $\mathrm{f}(x) = 4x^3 + kx + 6$, where k is a constant. When $\mathrm{f}(x)$ is divided by $(x-2)$, the remainder is 42. Use the remainder theorem to find the value of k. Hence find a root of $\mathrm{f}(x) = 0$. **[4]**

11. The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $\mathrm{p}(x)$ is divided by $x+2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x-3$ is a factor of $\mathrm{p}(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

12. $$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b.

(5)

13. Two cubic polynomials are defined by

$$f(x) = x^3 + (a - 3)x + 2b, \quad g(x) = 3x^3 + x^2 + 5ax + 4b,$$

where a and b are constants.

(i) Given that $f(x)$ and $g(x)$ have a common factor of $(x - 2)$, show that $a = -4$ and find the value of b. **[6]**

(ii) Using these values of a and b, factorise $f(x)$ fully. Hence show that $f(x)$ and $g(x)$ have two common factors. **[5]**

14. The cubic polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 17x + 6$.

(i) Find the remainder when $f(x)$ is divided by $(x - 3)$. **[2]**

(ii) Given that $f(2) = 0$, express $f(x)$ as the product of a linear factor and a quadratic factor. **[4]**

(iii) Determine the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

15. **(a) (i)** Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x)$ in the form $(x-3)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

16.
$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$

(2)

(b) factorise $f(x)$ completely.

(4)