Year 12 Lecture Series
Topic 8 – Exponentials & Logarithms

Inspired Learning

Mr A S Gill

Topic 8 – Exponentials & Logarithms

- Log laws

- Solving logarithmic equations

- Natural logs and 'e'

- Logarithmic graphs

Prerequisites:
Topic 1 – Algebra
Topic 2 – Coordinate Geometry

Topic 8 – Exponentials & Logarithms

$$a^x = b \iff \log_a b = x$$

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Logarithms

Write these in $\log form$:

$a)\ 5^3 = 125$

$b)\ 10^4 = 10000$

$c)\ a^3 = 8$

$d)\ 64 = 8^2$

$e)\ x^n = 10$

Evaluate the following (no calculator)

$a)\ \log_7 49$

$b)\ \log_2 32$

$c)\ \log_4 1$

$d)\ \log_2 \left(\dfrac{1}{2}\right)$

$e)\ \log_x x^3$

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Logarithmic laws

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Logarithmic laws

$\log ab =$

$\log \dfrac{a}{b} =$

$\log a^b =$

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Logarithmic laws

Express these as a single logarithm:

a) $\log 3 + \log 7$

b) $3 \log 4$

c) $2 \log 11 - \log 4$

d) $3 \log 5 + 4 \log 6 - 3 \log 10$

Express these in terms of separate logs $(a, b, and\ c)$:

a) $\log a^3 b$

b) $\log bc^2$

c) $\log \dfrac{ab^2}{c^3}$

d) $\log \dfrac{a^4 b^3}{\sqrt{c}}$

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Given that $a > 0$, state the values of

a $\log_a 1$,

b $\log_a (a^3)^6$,

c $\log_a \sqrt{a}$.

[MEI, GCE Mathematics, C2, Jan 2012]

Given that $\log_3 x = a$, find in terms of a,

a $\log_3 (9x)$

b $\log_3 \left(\dfrac{x^5}{81} \right)$

giving each answer in its simplest form.

c Solve, for x,

$$\log_3 (9x) + \log_3 \left(\frac{x^5}{81} \right) = 3$$

giving your answer to 4 significant figures.

[EDEXCEL, GCE Mathematics, C2 1R, June 2013]

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Solving Equations

Previously:

$2^x = 32$

Now:

$2^x = 37$

$3^{2x+1} = 81$

$3^{2x+1} = 850$

$2^{x+1} = 3^{3x-2}$

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Use logarithms to solve the equation $3^{x+1} = 5^{2x}$. Give your answer correct to 3 decimal places.   [MEI, GCE Mathematics, C2, June 2014]

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Solving Equations

$3^{2x} - 8(3^x) = 0$

a Find, to 3 significant figures, the value of x for which $5^x = 7$.

b Solve the equation $5^{2x} - 12(5^x) + 35 = 0$.

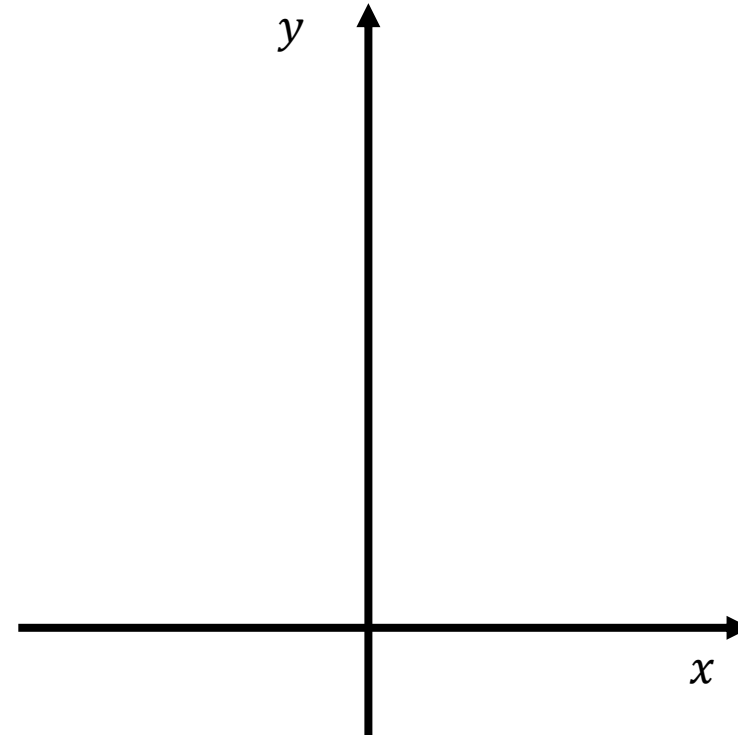[EDEXCEL, GCE Mathematics, C2, June 2008]

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Natural logs and 'e'

Exponential functions:

Graph transformations (*see topic* 1)

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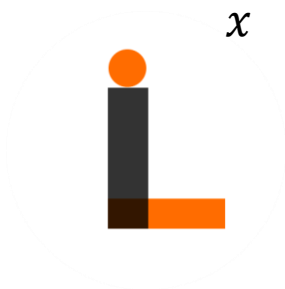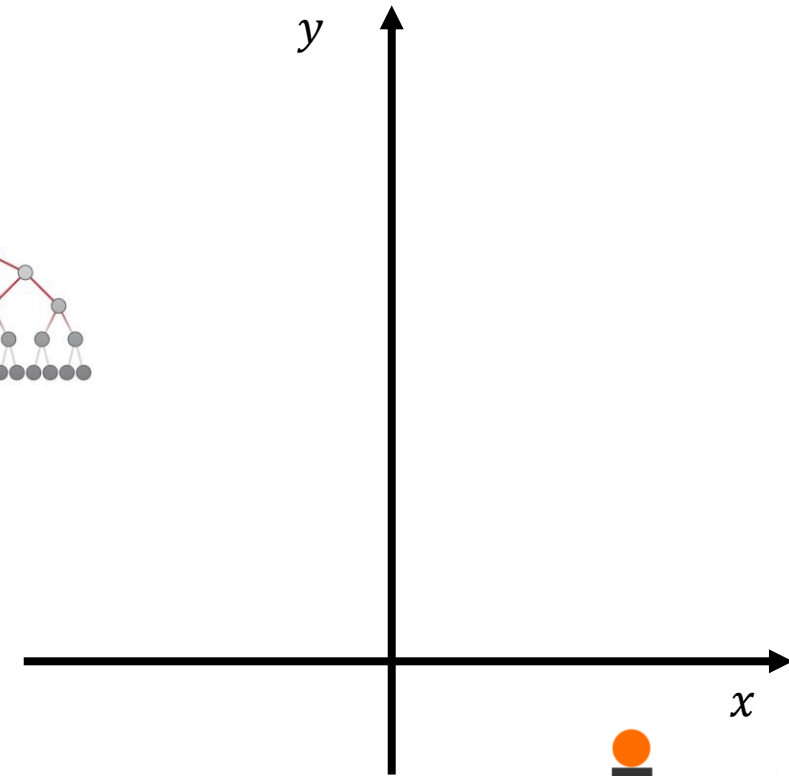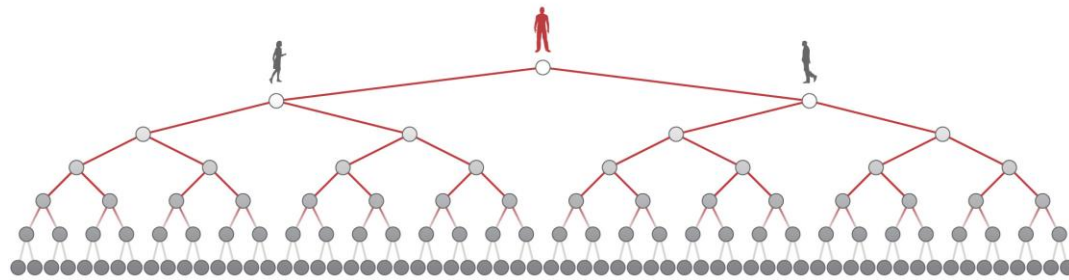
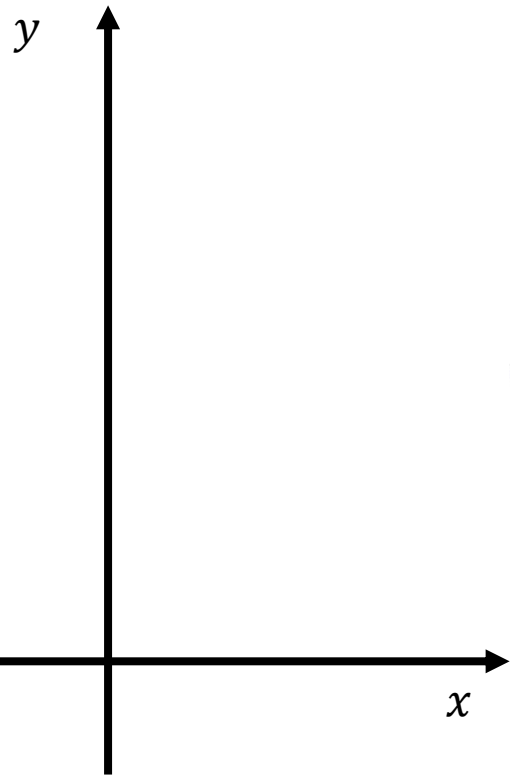*Solve the equations below*:
a) $e^x = 7$
b) $\ln(3x - 1) = 4$
c) $e^{2x} + 2e^x - 15 = 0$

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Exponential Growth & *Decay*:

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A substance is decaying exponentially. Its mass $(m, grams)$
after t years is given by:
$m = 200e^{-0.04t}$.
a) Find the value of m when $t = 50$
b) Find the value of t when $m = 50$.
c) What is the initial value of m?

A population of flies P is given by the formula $P = A\,e^{-kt}$, where t is the time in days measured from a time when $P = 2000$.

a Write down the value of A.

b Given that $P = 500$ when $t = 5$, show that $k = \frac{1}{5}\ln 4$.

c Find the value of P when $t = 8$ days.

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Solve the inequality $0.3^x < 0.02.$

The number of members of a social networking site is modelled by $m = 150e^{2t}$, where m is the number of members and t is time in weeks after the launch of the site.

b What is the significance of the integer 150 in the model?

c Find the week in which the model predicts that the number of members first exceeds 60 000.

[OCR, GCE Mathematics, Specimen Pure and Mechanics, 2017]

Logarithmic Graphs

$y = ab^x$

$\therefore \log y = \log(ab^x)$

$\log y = \log a + x \log b$

$y = \quad c \quad + \quad mx$

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The graph below shows $\log_{10} y$ against x. Find an equation relating $\log_{10} y$ and x, and hence find an equation relating x and y.

Use the graph below, which shows the relationship between $\log_{10} x$ and $\log_{10} y$, to find an equation connecting x and y.
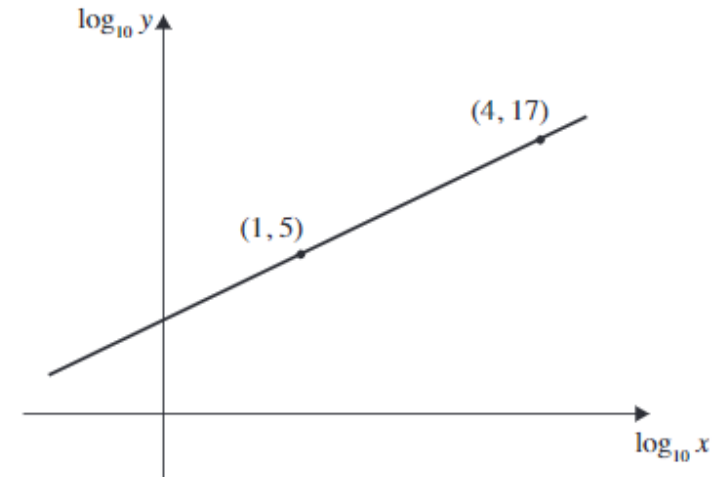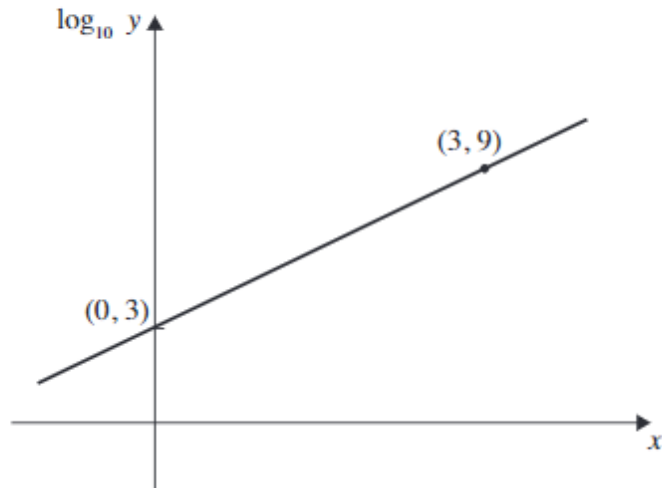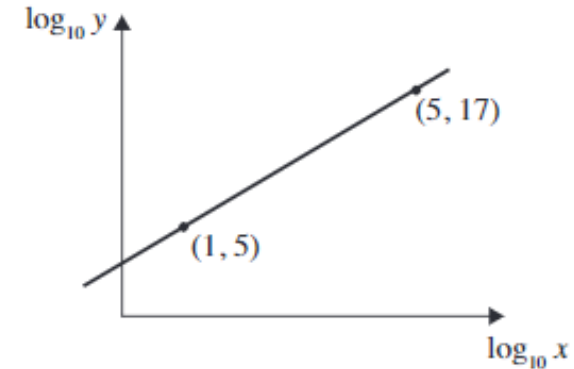
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It is believed that the variables x and y are connected by an equation in the form $y = kb^x$.

A plot of $\log_{10} y$ against x gives the coordinates $(1, 6)$ and $(3, 2)$.

Find the values of k and b.

The graph shows the relationship between $\log_{10} x$ and $\log_{10} y$.

Find y in terms of x.

[MEI, GCE Mathematics, C2, June 2012]

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The temperature of an object left to cool in a refrigerator set at 0°C can be modelled by an equation of the form $T = kb^{-x}$, where x is the time in hours, and T is the temperature in degrees celsius.

a Use the data given below to form an equation relating T and x, giving the values of any constants correct to two significant figures.

Time in hours, x	1	2	3	4	5
Temp in °C, T	68	29	14	6.4	2.9

b Use your equation to estimate the initial temperature of the object.

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Summary:

- *Logarithms are the inverse of exponents.*

- *You can use them to solve equations.*

- *There are certain laws that apply to logarithms.*

- *'e' is a constant and* $\ln$ *is* $\log_e$.

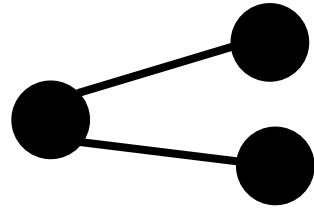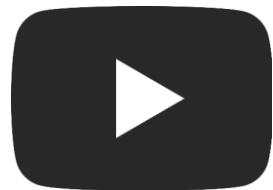- *We can model logarithmic graphs using linear plots.*

See you next week!

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