Year 12 Lecture Series
Topic 1 – Algebra & Functions

Inspired Learning
Mr A S Gill

Topic 1 – Algebra & Functions

Indices

$a^b \times a^c =$

$a^b \div a^c =$

$(a^b)^c =$

$a^0 =$

$a^{\frac{1}{b}} =$

$a^{-b} =$

$a^{\frac{c}{b}} =$

$a^{-\frac{c}{b}} =$

$a^b \times f^b =$

Topic 1 – Algebra & Functions

Indices

Evaluate the following:

1) $2^2 \times 2^2$

2) $27^{\frac{1}{3}}$

3) $(\sqrt{9})^3$

Topic 1 – Algebra & Functions

Indices

Evaluate the following:

1) $2^2 \times 2^2$
2) $27^{\frac{1}{3}}$
3) $(\sqrt{9})^3$

Simplify the following:

1) $x^3 \times x^4$
2) $\frac{a^{12}}{a^5}$
3) $(s^3)^3 \times (s^2)^9$
4) $(k^{\frac{1}{2}})^6$

Topic 1 – Algebra & Functions

Indices

Evaluate the following:

1) $2^2 \times 2^2$
2) $27^{\frac{1}{3}}$
3) $(\sqrt{9})^3$

Simplify the following:

1) $x^3 \times x^4$
2) $\frac{a^{12}}{a^5}$
3) $(s^3)^3 \times (s^2)^9$
4) $(k^{\frac{1}{2}})^6$

Simplify the following:

1) $(a^3)^3$
2) $c^7 \div c$
3) $2x^3 \times 5xy$
4) $10t^4u^2 \div 2t^2$
5) $12x^4y^2z \div 4x^3y^2$
6) $(36m^4n^2)^{\frac{1}{2}}$
7) $(8a^6b^3)^{\frac{1}{3}}$
8) $(16x^8y^{12})^{\frac{1}{4}} \times (3xy^3)$

Topic 1 – Algebra & Functions

Indices

$Solve\ the\ equation\ 2^n = 16$

Topic 1 – Algebra & Functions

Indices

$Solve\ the\ equation\ 2^n = 16$

$Solve\ the\ equation\ 3^{2x-1} = 27$

$Want\ a^x = a^y$

Topic 1 – Algebra & Functions

$Solve\ the\ equation\ x^{\frac{2}{3}} = \frac{1}{4}$

Topic 1 – Algebra & Functions

$Solve\ the\ equation\ x^{\frac{2}{3}} = \frac{1}{4}$

Find the value of n (as a fraction or whole number) in the following equations:

a $27^{n-1} = 3^6$ **b** $8^{2n+1} = 32^{n+1}$

Solve the following equations, showing your working clearly

d $2x^3 - 54 = 0$ **e** $5x^{-2} - 80 = 0$ **f** $2x^{\frac{2}{3}} - 1 = 17$

Topic 1 – Algebra & Functions

Surds

- *Irrational numbers*

Topic 1 – Algebra & Functions

Surds

- $Irrational\ numbers$
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b} \quad and \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Topic 1 – Algebra & Functions

Surds

Example 1

a $(5 - \sqrt{3})(7 + \sqrt{3})$

b $(\sqrt{2} + 1)(\sqrt{2} + 5)$

Topic 1 – Algebra & Functions

Surds

Example 1

a $(5 - \sqrt{3})(7 + \sqrt{3})$

b $(\sqrt{2} + 1)(\sqrt{2} + 5)$

Simplify the following:

$\sqrt{75}$:

$$\frac{\sqrt{27}}{\sqrt{3}}$$

$\sqrt{3} + \sqrt{12}$

Topic 1 – Algebra & Functions

Rationalising:

$\frac{10}{\sqrt{5}}$

$\frac{15}{\sqrt{3}}$

$\frac{16}{\sqrt{8}}$

$\frac{50\sqrt{2}}{\sqrt{10}}$

Topic 1 – Algebra & Functions

Rationalising:

$$\frac{2}{3+\sqrt{2}}$$

$$\frac{14+2\sqrt{5}}{3-\sqrt{5}}$$

Topic 1 – Algebra & Functions

Rationalising:

$$\frac{2}{3+\sqrt{2}}$$

$$\frac{14+2\sqrt{5}}{3-\sqrt{5}}$$

5 a Express $\frac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers.

b Express $\sqrt{18} - \sqrt{2}$ in simplified surd form.

[OCR, GCE Mathematics, C1, June 2010]

Topic 1 – Algebra & Functions

Polynomial Expressions

Quadratics:

- Factorising
- Formula
- CTS
- Discriminant

Topic 1 – Algebra & Functions

Polynomial Expressions

Quadratics:

- Factorising
- Formula
- CTS
- Discriminant

By completing the square, find the coordinates of the turning point of the curve $y = x^2 + 6x + 11$.

For the quadratic function $f(x) = -x^2 - 2x + 7$,

a write $f(x)$ in completed square form.

b hence find the line of symmetry and the coordinates of the vertex.

c sketch the curve $y = 7 - 2x - x^2$

Topic 1 – Algebra & Functions

Polynomial Expressions

Quadratics:

- **Factorising**
- **Formula**
- **CTS**
- **Discriminant**

Calculate the discriminant of $f(x) = 2x^2 + 11x + 5$ and hence state how many solutions there are to the equation $f(x) = 0$.

Find an inequality involving p such that the equation $px^2 + 2x + 1 = 0$ has no solutions.

Topic 1 – Algebra & Functions

Inequalities

Draw it out!!!

Topic 1 – Algebra & Functions

Inequalities

Solve $x^2 - 3x - 40 < 0$.

Solve $x^2 - 10x + 21 \geqslant 0$.

Topic 1 – Algebra & Functions

The equation $(p-1)x^2 + 4x + (p-5) = 0$, where p is a constant, has no real roots.

a Show that p satisfies $p^2 - 6p + 1 > 0$.

b Hence find the set of possible values of p.

[EDEXCEL, GCE Mathematics, C1, June 2015]

The equation $kx^2 + 4x + (5-k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(b) Hence find the set of possible values of k.

Topic 1 – Algebra & Functions

Simultaneous Equations

Solve $5x + 7y = 19$

and $3x + 2y = 7$

Topic 1 – Algebra & Functions

Simultaneous Equations

Solve $3x + y = 10$
and $x^2 + 2xy + 2y^2 = 17$

Solve the simultaneous equations $3x + y = 7$
$xy + x^2 = 6$

Topic 1 – Algebra & Functions

Simultaneous Equations

a Solve the simultaneous equations

$$y = 2x^2 - 3x - 5, \quad 10x + 2y + 11 = 0.$$

b What can you deduce from the answer to part a about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$?

[OCR, GCE Mathematics, C1, Jan 2013]

Topic 1 – Algebra & Functions

Sketching Graphs – Common Curves

Topic 1 – Algebra & Functions

Sketching Graphs

Sketch the graph of $y = x^2 - x - 2$.

Sketch the graph of $y = (x + 3)(x + 1)(x - 5)$.

Topic 1 – Algebra & Functions

(a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

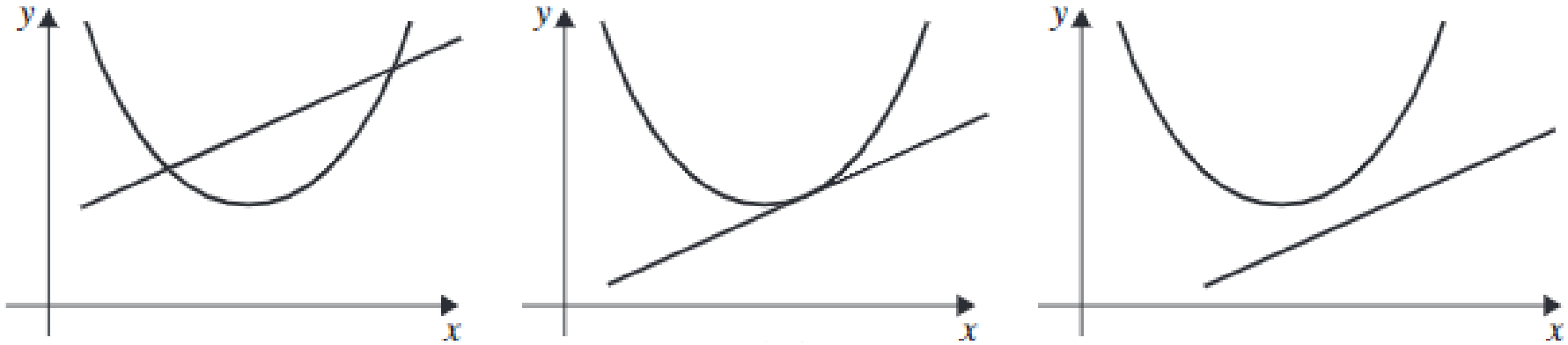
(4)

Topic 1 – Algebra & Functions

Sketching Graphs & Simultaneous Equations

Topic 1 – Algebra & Functions

Sketching Graphs & Simultaneous Equations



Topic 1 – Algebra & Functions

Sketching Graphs & Simultaneous Equations

Show that the line $y = x - 4$ is a tangent to the circle $x^2 + y^2 = 8$.

Topic 1 – Algebra & Functions

Sketching Graphs & Simultaneous Equations

Find the values of k such that the line $y = kx$ is a tangent to the curve $y = x^2 + 100$.

Topic 1 – Algebra & Functions

Graph Transformations

$y = f(x) \pm a$

$y = f(x \pm a)$

$y = -f(x)$

$y = f(-x)$

$y = af(x)$

$y = f(ax)$

Topic 1 – Algebra & Functions

Graph Transformations

$y = f(x) \pm a$

$y = f(x \pm a)$

$y = -f(x)$

$y = f(-x)$

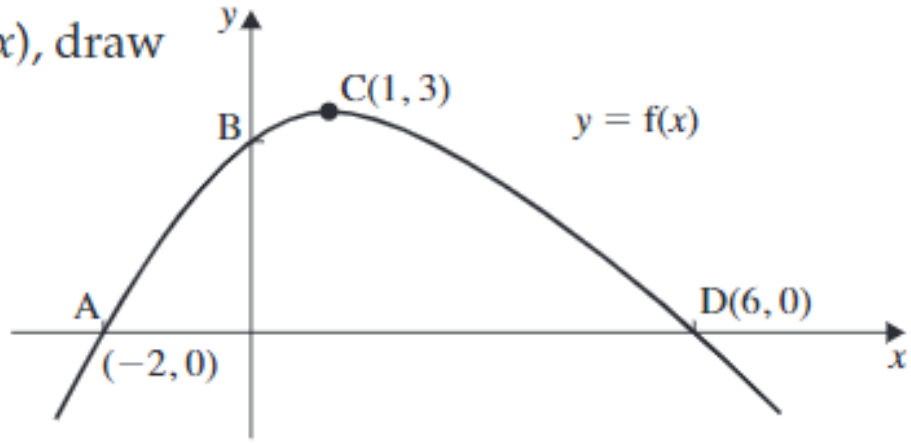
$y = af(x)$

$y = f(ax)$

From the sketch of $y = f(x)$, draw

a $y = f(x + 3)$,

b $y = f(3x)$



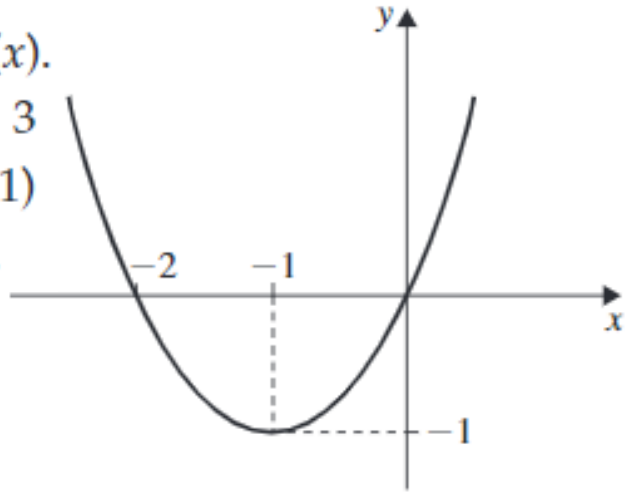
Topic 1 – Algebra & Functions

This is the sketch graph of $y = f(x)$.

a Sketch the graph of $y = f(x) + 3$

b Sketch the graph of $y = f(x + 1)$

c Sketch the graph of $y = -f(x)$.



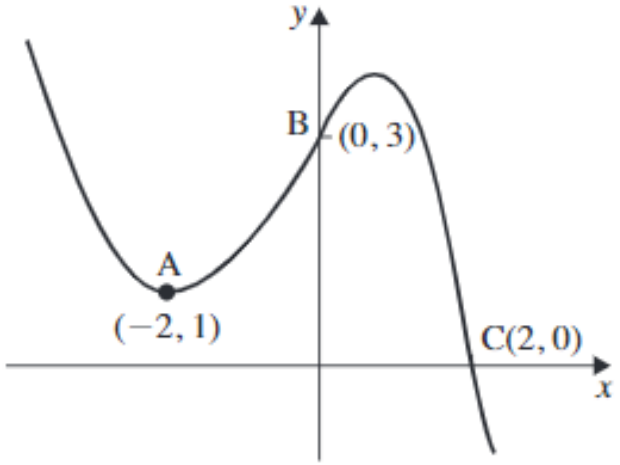
This is the sketch of $y = f(x)$ which passes through A, B, C.

Sketch the following curves, giving the new coordinates of A, B, C in each case.

a $y = -f(x)$

b $y = f(x - 2)$

c $y = f(2x)$



Topic 1 – Algebra & Functions

(a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

Topic 1 – Algebra & Functions

a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers.

b) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. [AQA, GCE Mathematics, C1, June 2011]