Inspired Learning

1.

(a) Evaluate $81^{\frac{3}{2}}$

(2)

(b) Simplify fully $x^2\left(4x^{-\frac{1}{2}}\right)^2$

(2)

2. Express each of the following in the form 4^n:

(i) $\frac{1}{16}$, [1]

(ii) 64, [1]

(iii) 8. [2]

3.

(a) Find the value of $8^{\frac{5}{3}}$

(2)

(b) Simplify fully $\dfrac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2}$

(3)

4. Solve the equations

(i) $3^m = 81$, **[1]**

(ii) $(36p^4)^{\frac{1}{2}} = 24$, **[3]**

(iii) $5^n \times 5^{n+4} = 25$. **[3]**

5.

(i) Express $\dfrac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers. **[3]**

(ii) Express $\sqrt{18} - \sqrt{2}$ in simplified surd form. **[2]**

6. (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(b) Express $(2-\sqrt{3})^2$ in the form $b+c\sqrt{3}$, where b and c are integers to be found.

(3)

7. Express $\dfrac{8+\sqrt{7}}{2+\sqrt{7}}$ in the form $a+b\sqrt{7}$, where a and b are integers. **[4]**

8. (a) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $m+n\sqrt{5}$, where m and n are integers. *(4 marks)*

(b) Express $\sqrt{45}+\dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. *(3 marks)*

9. Solve the equation

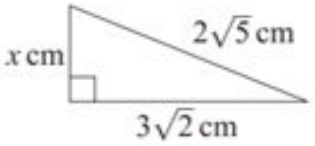
$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4)

10. (a) Express $\dfrac{5+\sqrt{7}}{3-\sqrt{7}}$ in the form $m+n\sqrt{7}$, where m and n are integers. *(4 marks)*

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x. *(3 marks)*

11. The equation

$$(p-1)x^2 + 4x + (p-5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p.

(4)

12. **(i)** Solve the equation $5 - 8x - x^2 = 0$, giving your answers in simplified surd form. **[3]**

(ii) Solve the inequality $5 - 8x - x^2 \leqslant 0$. **[2]**

(iii) Sketch the curve $y = (5 - 8x - x^2)(x + 4)$, giving the coordinates of the points where the curve crosses the coordinate axes. **[5]**

13. The quadratic equation

$$(2k-7)x^2 - (k-2)x + (k-3) = 0$$

has real roots.

(a) Show that $7k^2 - 48k + 80 \leqslant 0$. *(4 marks)*

(b) Find the possible values of k. *(4 marks)*

14. Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. **[5]**

15. Solve the equation $2x - 7x^{\frac{1}{2}} + 3 = 0$. [5]

16. Solve the equation $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$. **[5]**

17. **(i)** Express $5x^2 + 20x - 8$ in the form $p(x+q)^2 + r$. **[4]**

(ii) State the equation of the line of symmetry of the curve $y = 5x^2 + 20x - 8$. **[1]**

(iii) Calculate the discriminant of $5x^2 + 20x - 8$. **[2]**

(iv) State the number of real roots of the equation $5x^2 + 20x - 8 = 0$. **[1]**

18. Given that

$$5x^2 + px - 8 = q(x-1)^2 + r$$

for all values of x, find the values of the constants p, q and r. **[4]**

19. Solve the simultaneous equations

$$y = 2(x-2)^2, \qquad 3x + y = 26.$$ **[5]**

20. **(i)** Solve the simultaneous equations

$$y = 2x^2 - 3x - 5, \qquad 10x + 2y + 11 = 0.$$ **[5]**

(ii) What can you deduce from the answer to part **(i)** about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$? **[1]**

21. Solve each of the following inequalities:

(a) $2(4-3x) > 5 - 4(x+2)$; *(2 marks)*

(b) $2x^2 + 5x \geqslant 12$. *(4 marks)*

22. Solve the inequality $3x^2 + 10x + 3 > 0$. **[3]**

23. Solve the inequalities

(i) $-9 \leqslant 6x + 5 \leqslant 0$, **[3]**

(ii) $6x + 5 < x^2 + 2x - 7$. **[5]**

24.

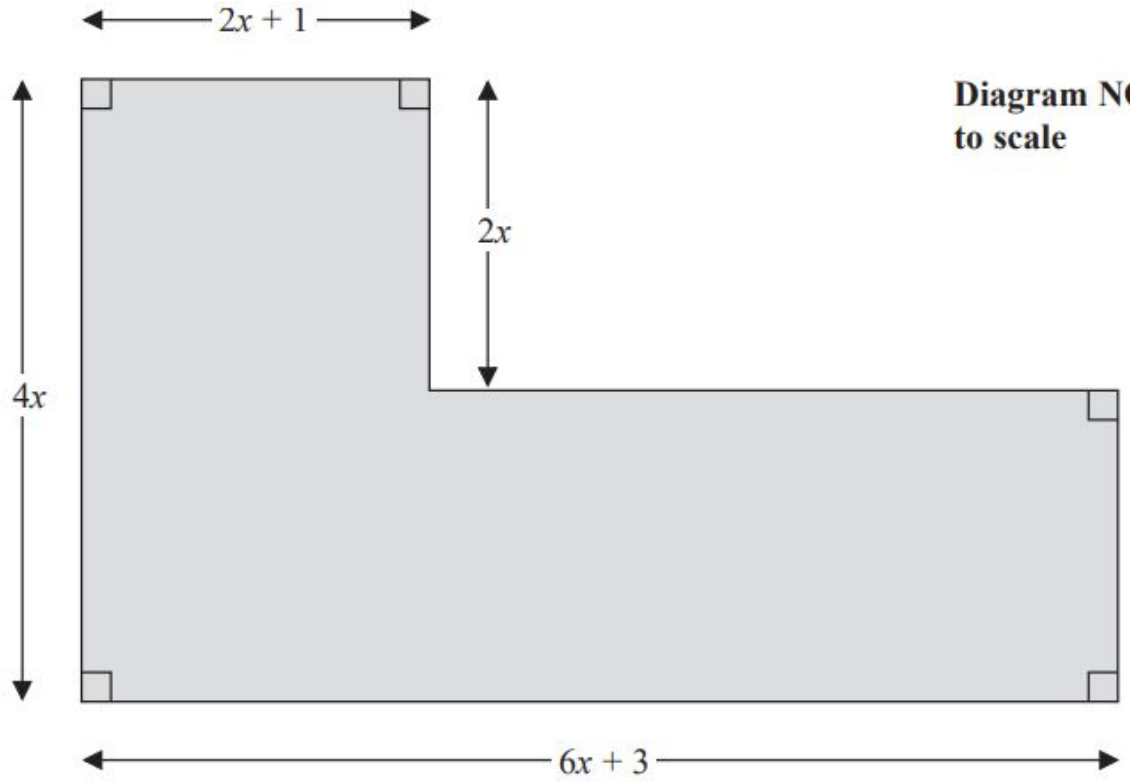


Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 40 m,

(a) show that $x > 1.7$

(3)

Given that the area of the garden is less than 120 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of the possible values of x.

(1)

25.

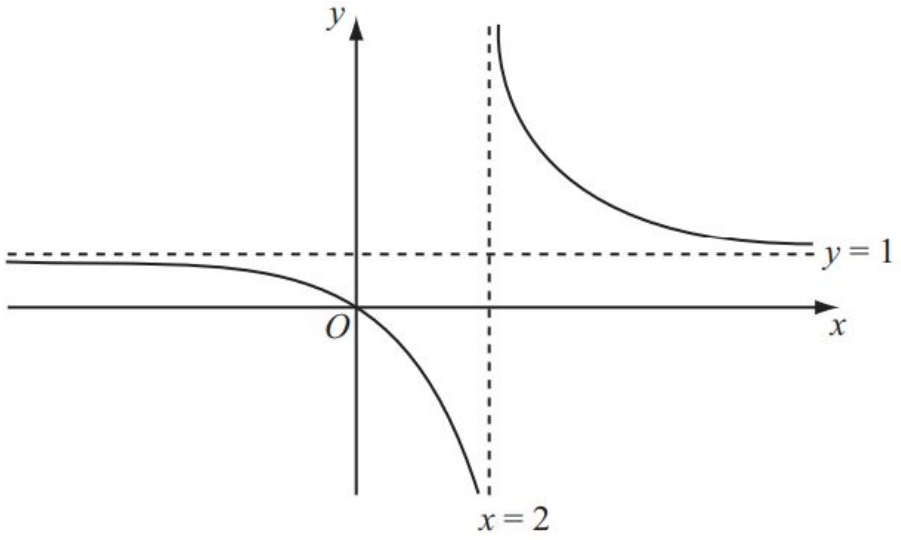


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes.

(4)

26.

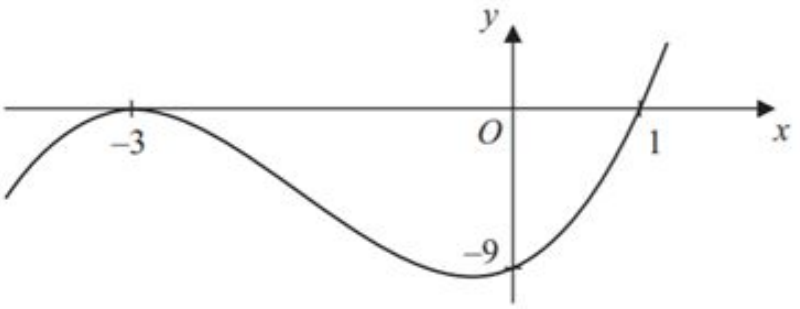


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (x + 3)^2 (x - 1), \qquad x \in \mathbb{R}.$$

The curve crosses the x-axis at $(1, 0)$, touches it at $(-3, 0)$ and crosses the y-axis at $(0, -9)$

(a) In the space below, sketch the curve C with equation $y = f(x + 2)$ and state the coordinates of the points where the curve C meets the x-axis.

(3)

(b) Write down an equation of the curve C.

(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C meets the y-axis.

(2)

27. **(i)** Sketch the curve $y = -x^3$. **[2]**

(ii) The curve $y = -x^3$ is translated by 3 units in the positive x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = -x^3$ to the curve $y = -5x^3$. **[2]**

28. **(a)** Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers. *(3 marks)*

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) State the equation of the line of symmetry of the curve. *(1 mark)*

(iii) Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*

29. The point $P\ (1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \frac{2}{x}$.

On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)