Inspired Learning

1.

$(3-2x)^5 = 243,\quad \ldots\ldots + 5\times(3)^4(-2x) = -810x\quad \ldots\ldots$	B1, B1
$+\dfrac{5\times 4}{2}(3)^3(-2x)^2 = \qquad +1080x^2$	M1 A1 (4)
	[4]

2.

$81x^4 - 216x^3 + 216x^2 - 96x + 16$	4 [4]	**M3** for 4 terms correct or for all coefficients correct except for sign errors or for correct answer seen then further 'simplified' or for all terms correct eg seen in table but not combined or **M2** for 3 terms correct or for correct expansion seen without correct evaluation of coefficients [if brackets missing in elements such as $(3x)^2$ there must be evidence from calculation that $9x^2$ has been used] or **M1** for 1 4 6 4 1 row of Pascal's triangle seen	condone eg $+(-96x)$ or $+-96x$ instead of $-96x$ any who multiply out instead of using binomial coeffts: look at their final answer and mark as per main scheme if 3 or more terms are correct, otherwise M0 binomial coefficients such as ${}^4C_2$ or $\binom{4}{2}$ are not sufficient – must show understanding of these symbols by at least partial evaluation;

3.

$\left(1+\frac{x}{3}\right)^6=1+\binom{6}{1}\frac{x}{3}+\binom{6}{2}\left(\frac{x}{3}\right)^2+\binom{6}{3}\left(\frac{x}{3}\right)^3$ ...			
$=(1+)\ 2x$	B1		$a=2$. Condone '$2x$'
$+\frac{6!}{4!2!}\left(\frac{x}{3}\right)^2+\frac{6!}{3!3!}\left(\frac{x}{3}\right)^3$	M1		Either (1 6) 15 20 seen or $\binom{6}{2}$, $\binom{6}{3}$ written (PI) in terms of factorials (OE)
$=(1+2x)$ $+\frac{15}{9}x^2+\frac{20}{27}x^3$ ($a=2$)			
$b=\frac{5}{3}$, $c=\frac{20}{27}$	A1		$b=\frac{5}{3}$ (or $1\frac{2}{3}$). Condone $\ldots+\frac{5}{3}x^2$
	A1	4	$c=\frac{20}{27}$. Condone $\ldots+\frac{20}{27}x^3$ Accept equivalent recurring decimals Ignore terms with higher powers of x SC If A0A0 award A1 for either $+15\frac{x^2}{9}$, $+20\frac{x^3}{27}$ seen or $+\frac{15x^2}{9}$, $+\frac{20x^3}{27}$ seen
Total		8	

4.

(a)	$(1-x)^3 = 1-3x+3x^2-x^3$	M1 A1	2	3 terms correct or 1 (±)3 (±)3 (±)1 seen All correct
(b)	$(1+y)^4 = 1+4y+6y^2+4y^3+y^4$	M1 A1		4 terms correct, accept unsimplified All 5 terms correct and simplified at some stage
	$(1+y)^4-(1-y)^3=$ $(4y+3y)+(6y^2-3y^2)+(4y^3+y^3)+y^4$ $= 7y+3y^2+5y^3+y^4$ (as required with p=3 and q=5)	A2,1	4	A2 Be convinced as part answer is given (A1 for three terms found correctly or if found correct values for p and q but did not show $7y+y^4$.)
(c)	$\int \left[\left(1+\sqrt{x}\right)^4-\left(1-\sqrt{x}\right)^3\right] \, dx =$			
	$\int\left(7\sqrt{x}+3x+5x\sqrt{x}+x^2\right) \, dx$	M1		Use of part (**b**)….$y \to \sqrt{x}$ OE before any integration
	$\int\left(7x^{0.5}+3x+5x^{1.5}+x^2\right) \, dx$			
	$= \frac{7x^{1.5}}{1.5}+\frac{3x^2}{2}+\frac{5x^{2.5}}{2.5}+\frac{x^3}{3} \; (+c)$	m1		Correct integration of an x^k term where k is non-integer
	$= \frac{14}{3}x^{1.5}+\frac{3}{2}x^2+2x^{2.5}+\frac{1}{3}x^3 \; (+c)$	A2,1F	4	Coeffs simplified; condone absent $(+c)$ Ft on c's p and q ie 2nd term $+\frac{p}{2}x^2$ and 3rd term is $+\frac{2q}{5}x^{2.5}$. (A1F for three of these four ft terms or for four correct ft terms unsimplified)
	Total		**10**	

5.

6000	4	**M3** for $15 \times 5^2 \times 2^4$; or **M2** for two of these elements correct with multiplication or all three elements correct but without multiplication (e.g. in list or with addition signs); or **M1** for 15 soi or for 1 6 15 ... seen in Pascal's triangle; SC2 for $20000[x^3]$	condone inclusion of x^4 eg $(2x)^4$; condone omission of brackets in $2x^4$ if 16 used; allow **M3** for correct term seen (often all terms written down) but then wrong term evaluated or all evaluated and correct term not identified; $15 \times 5^2 \times (2x)^4$ earns **M3** even if followed by $15 \times 25 \times 2$ calculated; no MR for wrong power evaluated but SC for fourth term evaluated

6.

Answer	Marks	Guidance	
$(3+2x)^5 = 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	M1*	Attempt expansion – products of powers of 3 and $2x$	Must attempt at least 5 terms. Each term must be an attempt at a product, including binomial coeffs if used. Allow M1 for no, or incorrect, binomial coeffs. Powers of 3 and $2x$ must be intended to sum to 5 within each term (allow slips if intention correct). Allow M1 even if powers used incorrectly with the $2x$ ie $2x^3$ not $(2x)^3$ can get M1. Allow M1 for powers of $^2/_3x$ from expanding $k(1 + {}^2/_3x)^5$, any k (allow if powers only applied to x and not $^2/_3$).
	M1d*	Attempt to use correct binomial coefficients	At least 5 correct from 1, 5, 10, 10, 5, 1 - allow missing or incorrect (but not if raised to a power). May be implied rather than explicit. Must be numerical eg $^5C_1$ is not enough. They must be part of a product within each term. The coefficient must be used in an attempt at the relevant term ie 5 x 3^3 x $(2x)^2$ is M0. Allow M1 for correct coefficients from expanding $k(1 + {}^2/_3x)^5$, any k.
	A1	Obtain at least four correct simplified terms	Either linked by '+' or as part of a list.
	A1	Obtain fully correct expansion	With all coefficients simplified. Terms must be linked by '+' and not just commas.
	[4]		**SR for reasonable expansion attempt:** M2 for attempt involving all 5 brackets resulting in a quintic with at most one term missing A1 for four correct, simplified, terms A1 for fully correct, simplified, expansion

7.

(i)	10 cao	1 [1]		
(ii)	$-720\ [x^3]$	4 [4]	B3 for 720 $[x^3]$ or for $10 \times 9 \times -8\ [x^3]$ or M2 for $10 \times 3^2 \times (-2)^3$ oe or ft from (i) or M1 for two of these three elements correct or ft; condone x still included	condone $-720\,x$ etc allow equivalent marks for the x^3 term as part of a longer expansion eg M2 for $3^5\left(...10\times\left(\frac{-2}{3}\right)^3...\right)$ or M1 for $10\times\left(\frac{-2}{3}\right)^3$ etc

8.

(a) $(1+ax)^{10} = 1 + 10ax......$ (Not unsimplified versions)	B1	
$+\frac{10\times 9}{2}(ax)^2 + \frac{10\times 9\times 8}{6}(ax)^3$ Evidence from one of these terms is sufficient	M1	
$+45(ax)^2, +120(ax)^3$ or $+45a^2x^2, +120a^3x^3$	A1, A1	(4)
(b) $120a^3 = 2\times 45a^2$ $a = \frac{3}{4}$ or equiv. $\left(\text{e.g.} \frac{90}{120}, 0.75\right)$ Ignore $a = 0$, if seen	M1 A1	(2)
		6

9.

$\{(3+bx)^5\} = (3)^5 + {}^5C_1(3)^4(bx) + {}^5C_2(3)^3(bx)^2 + \ldots$ $= 243 + 405bx + 270b^2x^2 + \ldots$	243 as a constant term seen. $405bx$ $({}^5C_1 \times \ldots \times x)$ or $({}^5C_2 \times \ldots \times x^2)$ $270b^2x^2$ or $270(bx)^2$	B1 B1 M1 A1	 [4]
$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$ So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation. $b = 3$ (Ignore $b = 0$, if seen.)	M1 A1	 [2] 6

10.

	Solution	Marks	Total	Comments
(a)(i)	$\{(2+y)^3 =\}\ 8 + 12y + 6y^2 + y^3$	M1		At least 3 terms simplified and correct
		A1	2	All correct
(ii)	$(2+x^{-2})^3 = 8 + 12x^{-2} + 6(x^{-2})^2 + (x^{-2})^3$	M1		A replacement of y by x^{-2} in c's (a)(i) working. PI
	$(2-x^{-2})^3 = 8 - 12x^{-2} + 6(x^{-2})^2 - (x^{-2})^3$	A1F		Ft one incorrect coefficient in (a)(i) expansion.
	$(2+x^{-2})^3 + (2-x^{-2})^3 = 16 + 12x^{-4}$	A1	3	CSO Be convinced.
				SC2 for a fully correct solution, not using 'Hence'

11.

(i)	$20 \times 4^3 \times a^3 = 160$ $1280a^3 = 160$ $a^3 = \frac{1}{8}$ $a = \frac{1}{2}$	**M1**	Attempt relevant term	Must be an attempt at a product involving a binomial coeff of 20 (not just ${}^6C_3$ unless later seen as 20), 4^3 and an intention to cube ax (but allow for ax^3) Could come from $4^6(1 + {}^{ax}/_4)^6$ as long as done correctly Ignore any other terms if fuller expansion attempted
		A1	Obtain correct $1280a^3$, or unsimplified equiv	Allow $1280a^3x^3$, or $1280(ax)^3$, but not $1280ax^3$ unless a^3 subsequently seen, or implied by working
		M1	Equate to 160 and attempt to solve for a	Must be equating coeffs – allow if x^3 present on both sides (but not just one) as long as they both go at same point Allow for their coeff of x^3, as long as two, or more, parts of product are attempted eg $20ax^3$ / $64ax^3$ Allow M1 for $1280a = 160$ (giving $a = 0.125$) M0 for incorrect division (eg giving $a^3 = 8$)
		A1	Obtain $a = \frac{1}{2}$	Allow 0.5, but not an unsimplified fraction Answer only gets full credit, as does T&I **SR: max of 3 marks** for $a = 0.5$ from incorrect algebra, eg $1280ax^3 = 160$, so $a = 0.5$ would get M1A1(implied)B1
		[4]		

(ii)	$4^6 + 6 \times 4^5 \times \frac{1}{2} = 4096 + 3072x$	**B1**	State 4096	Allow 4^6 if given as final answer Mark final answer – so do not isw if a constant term is subsequently added to 4096 from an incorrect attempt at second term eg using sum rather than product
		B1FT	State $3072x$, or $(6144 \times \text{their } a)x$	Must follow a numerical value of a, from attempt in part **(i)** Must be of form kx so just stating coeff of x is B0 Mark final answer B2 can still be awarded if two terms are not linked by a '+' sign – could be a comma, 'and', or just two separate terms **SR: B1** can be awarded if both terms seen as correct, but then 'cancelled' by a common factor
		[2]		

12.

3. (a)	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\{(2-3x)^6\} = (2)^6 + \underline{^6C_1}(2)^5(-3\underline{x}) + \underline{^6C_2}(2)^4\underline{(-3x)^2} + ...$		<u>M1</u>
	M1: $({}^6C_1 \times ... \times x)$ or $({}^6C_2 \times ... \times x^2)$. For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.		
	${}^6C_1 2^5 - 3x + {}^6C_2 2^4 - 3x^2 + ...$ Scores M0 unless later work implies a correct method		
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
			[4]
(a) Way 2	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\left(1 - \frac{3}{2}x\right)^6 = 1 + \underline{^6C_1}\left(\frac{-3}{2}\underline{x}\right) + \underline{^6C_2}\left(\frac{-3}{2}\underline{x}\right)^2 + ...$	M1: $({}^6C_1 \times ... \times x)$ or $({}^6C_2 \times ... \times x^2)$. For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.	<u>M1</u>
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
(b)	Candidate writes down $\left(1 + \frac{x}{2}\right) \times$ (their part (a) answer, at least up to the term in x). (Condone missing brackets) $\left(1+\frac{x}{2}\right)(64 - 576x + ...)$ or $\left(1+\frac{x}{2}\right)(64 - 576x + 2160x^2 + ...)$ or $\left(1+\frac{x}{2}\right)64 - \left(1+\frac{x}{2}\right)576x$ or $\left(1+\frac{x}{2}\right)64 - \left(1+\frac{x}{2}\right)576x + \left(1+\frac{x}{2}\right)2160x^2$ or $64 + 32x, -576x - 288x^2,\ 2160x^2 + 1080x^3$ are fine.		M1
	$= 64 - 544x + 1872x^2 + ...$	A1: At least 2 terms correct as shown. (Allow $+ -544x$ here) A1: $64 - 544x + 1872x^2$ The terms can be "listed" rather than added. Ignore any extra terms.	A1A1
			[3]
			Total 7
	SC: If a candidate expands in descending powers of x, only the M marks are available		
	e.g. $\{(2-3x)^6\} = (-3x)^6 + \underline{^6C_1}(2)^1\underline{(-3x)^5} + \underline{^6C_2}(2)^2\underline{(-3x)^4} + ...$		

13.	(a)	$\left(1+\frac{4}{x}\right)^2 = 1+\frac{8}{x}+\frac{16}{x^2}$ (or $1+8x^{-1}+16x^{-2}$)	B1	1	Unsimplified equivalent answers, e.g. $1+\frac{4}{x}+\frac{4}{x}+\left(\frac{4}{x}\right)^2$ etc. must be correctly simplified in part (c) to one of the two forms in 'solution' to retrospectively score the B1 here
	(b)	$\left(1+\frac{x}{4}\right)^8 = \{1+\}\binom{8}{1}\left(\frac{x}{4}\right)+\binom{8}{2}\left(\frac{x}{4}\right)^2+\binom{8}{3}\left(\frac{x}{4}\right)^3+\ldots$	M1		Any valid method. PI by a correct value for either a or b or c
		$=\{1+\}2x+\frac{7}{4}x^2+\frac{7}{8}x^3+\ldots$	A1A1A1		A1 for each of a, b, c
		$\{a=2,\ b=1.75\ \text{OE},\ c=0.875\ \text{OE}\}$		4	**SC** $a=8,\ b=28,\ c=56$ or $a=32,\ b=448,\ c=3584$ either explicitly or within expn (M1A0)
	(c)	$(1+\frac{8}{x}+\frac{16}{x^2})\left(1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3\right)$	M1		Product of c's two expansions either stated explicitly or used
		x terms from expansion of $\left(1+\frac{4}{x}\right)^2\left(1+\frac{x}{4}\right)^8$ are ax and '8'bx and '16'cx	m1		Any **two** of the three, **ft** from products of non-zero terms using c's two expansions. May just use the coefficients.
		ax + '8'bx + '16'cx	A1F		Ft on c's non-zero **values** for a, b **and** c and also ft on c's non-zero coeffs. of $1/x$ and $1/x^2$ in part (a). Accept x's missing i.e. sum of coeffs. PI by the correct final answer.
		Coefficient of x is $2+14+14=30$	A1	4	OE Condone answer left as $30x$. Ignore terms in other powers of x in the expansion.
		Total		**9**	

14.

(i)		$(2+5x)^6 = 64 + 960x + 6000x^2$	M1	Attempt at least first 2 terms– products of binomial coeff and correct powers of 2 and $5x$	Must be clear intention to use correct powers of 2 and $5x$ Binomial coeff must be 6 soi; ${}^6C_1$ is not yet enough Allow BOD if 6 results from ${}^6/_1$ Allow M1 if expanding $k(1+{}^5/_2x)^6$, any k
			A1	Obtain $64 + 960x$	Allow 2^6 for 64 Allow if terms given as list rather than linked by '+'
			M1	Attempt 3rd term – product of binomial coeff and correct powers of 2 and $5x$	Allow M1 for $5x^2$ rather than $(5x)^2$ Binomial coeff must be 15 soi; ${}^6C_2$ is not yet enough Allow M1 if expanding $k(1+{}^5/_2x)^6$, any k $1200x^2$ implies M1, as long as no errors seen (including no working shown)
			A1 [4]	Obtain $6000x^2$	A0 if an otherwise correct expansion is subsequently spoiled by attempt to simplify eg $4 + 60x + 375x^2$ **If expanding brackets:** Mark as above, but must consider all 6 brackets for the M marks (allow irrelevant terms to be discarded)
(ii)		$(9 + 6cx \ldots)(64 + 960x + \ldots)$	M1*	Expand first bracket and attempt at least one relevant product	Expansion of first bracket does not have to be correct, but must be attempted so M0 if using $(3+cx)(64+960x\ldots)$ No need to see third term in expansion of first bracket Must then consider a product and not just use $6c + 960$ Expansion could include irrelevant / incorrect terms Using an incorrect expansion associated with part (i) can get M1 M1
		$(9 \times 960) + (6c \times 64) = 4416$ $8640 + 384c = 4416$ $384c = -4224$	M1d*	Equate sum of the two relevant terms to 4416 and attempt to solve for c	Must now consider just the two relevant terms M0 if additional terms, even if error has resulted in kx BOD if presence of x is inconsistent within equation
		$c = -11$	A1 [3]	Obtain $c = -11$	A0 for $c = -11x$

16.

identifying term as $20(2x)^3\left(\frac{5}{x}\right)^3$ oe	M3	condone lack of brackets; M1 for $[k](2x)^3\left(\frac{5}{x}\right)^3$ soi (eg in list or table), condoning lack of brackets and M1 for $k = 20$ or eg $\frac{6\times5\times4}{3\times2\times1}$ or for 1 6 15 20 15 6 1 seen (eg Pascal's triangle seen, even if no attempt at expansion) and M1 for selecting the appropriate term (eg may be implied by use of only $k = 20$, but this M1 is not dependent on the correct k used)
20 000	A1 **[4]**	or B4 for 20 000 obtained from multiplying out $\left(2x+\frac{5}{x}\right)^6$ allow SC3 for 20000 as part of an expansion

17. (i)	243	B1	1	State 243, or 3^5
(ii)	2nd term = 5 x 3^4 x (kx) = $405kx$ 3rd term = 10 x 3^3 x $(kx)^2$ = $270k^2x^2$ $405k = 270k^2 \Rightarrow k = 1.5$	B1		Obtain $405k$ as coeff of x
		M1		Attempt coeff of x^2
		A1		Obtain $270k^2$
		M1		Equate coefficients and attempt to solve for k
		A1	5	Obtain $k = 1.5$ (ignore any mention of $k = 0$)
(iii)	10 x 3^2 x 1.5^3 = 303.75	M1		Attempt 10 x 3^2 x k^3
		A1	2	Obtain 303.75 (allow $303.75x^3$)
			8	

18.	(a)	Binomial seen or implied	M1	by use of table or ${}^9C_6$ or $(\frac{2}{3})^p(\frac{1}{3})^q$ $(p + q = 9)$	Eg 0.6228 seen
		0.6228 – 0.3497	M1	${}^9C_6(\frac{1}{3})^3(\frac{2}{3})^6$	
		= 0.273 (3 sf)	A1	$\frac{1792}{6561}$	
			[3]		